Further Math Mechanics (Paper 3) Topical 2016-2021

Compiled By A Fellow Student

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Assalam-o-Alaikum,
So basically while doing AS I was unable to find any topical for FM that was not behind a paywall or easily accessible and due to that I was unable to practice much for the paper. Therefore, after my P1 paper I felt like I should compile these workbook (P1 And Mechanics) so that others may have an easier time practicing and exploring this interesting/unique/difficult/depression inducing subject.

Oh and as only a student compiled this workbook there might be some errors. I have tried my best to eliminate them but sorry if you find any.

One last thing... most of the topics in further mechanics were previously in M2 of the other maths (9709) and the syllabus content matched word to word with the new further mechanics syllabus hence, several questions are taken from there.

A fellow student,
Craftear/AN

Craftear

Projectiles

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Q1

7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $100\,\text{m}\,\text{s}^{-1}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. The two times at which P's height above the plane is H m differ by 10 s.

(a) Find the value of H. [5]

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(b) Find the magnitude and direction of the velocity of P one second before it strikes the plane. [4]

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Q2

7 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range R in terms of u, θ and g. [2]

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(b) Deduce an expression for the maximum height H, in terms of u, θ and g. [2]

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It is given that $R = \frac{4H}{\sqrt{3}}$.

(c) Show that $\theta = 60°$. [1]

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It is given also that $u = \sqrt{40}\,\text{m}\,\text{s}^{-1}$.

(d) Find, by differentiating the equation of the trajectory or otherwise, the set of values of x for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

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Q3

5 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. Its initial speed is $u\,\mathrm{m\,s^{-1}}$ and its angle of projection is $\sin^{-1}(\frac{4}{5})$ above the horizontal. At time 8 s after projection, P is at the point A. At time 32 s after projection, P is at the point B. The direction of motion of P at B is perpendicular to its direction of motion at A.

Find the value of u. [7]

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Q4

1 A particle is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane. The particle moves freely under gravity.

(a) Write down the horizontal and vertical components of the velocity of the particle at time T after projection. [2]

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At time T after projection, the direction of motion of the particle is perpendicular to the direction of projection.

(b) Express T in terms of u, g and α. [2]

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(c) Deduce that $T > \frac{u}{g}$. [1]

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Q5

1 A particle P is projected with speed u at an angle of 30° above the horizontal from a point O on a horizontal plane and moves freely under gravity. The particle reaches its greatest height at time T after projection.

Find, in terms of u, the speed of P at time $\frac{2}{3}T$ after projection. [5]

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Q6

6 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The direction of motion of P makes an angle α above the horizontal when P first reaches three-quarters of its greatest height.

(a) Show that $\tan\alpha = \frac{1}{2}\tan\theta$. [6]

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(b) Given that $\tan\theta = \frac{4}{3}$, find the horizontal distance travelled by P when it first reaches three-quarters of its greatest height. Give your answer in terms of u and g. [4]

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Q7

5 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

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The point Q is the highest point on the trajectory of P in the case where $\alpha = 45°$.

(b) Show that the x-coordinate of Q is $\frac{u^2}{2g}$. [3]

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(c) Find the other value of α for which P would pass through the point Q. [4]

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Q8

5 A particle P is projected with speed $u\,\text{m s}^{-1}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t s are denoted by x m and y m respectively.

(a) Starting from the equation of the trajectory given in the List of formulae (MF19), show that

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta).$$ [1]

When $\theta = \tan^{-1}2$, P passes through the point with coordinates (10, 16).

(b) Show that there is no value of θ for which P can pass through the point with coordinates (18, 30). [6]

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Q9

1 A particle is projected with speed $24\ \text{m s}^{-1}$ at an angle of 30° above the horizontal. Find the speed and direction of motion of the particle at the instant 4 s after projection. [5]

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Q10

3 A small ball is projected from a point O on horizontal ground. At time t s after projection the horizontal and vertically upwards displacements of the ball from O are x m and y m respectively, where $x = 4t$ and $y = 6t - 5t^2$.

(i) Find the equation of the trajectory of the ball. [2]

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(ii) Hence or otherwise calculate the angle of projection of the ball and its initial speed. [4]

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Q11

2 A particle is projected with speed V m s^{-1} at an angle of $\theta°$ above the horizontal. At the instant 4 s after projection the speed of the particle is 16 m s^{-1} and its direction of motion is 30° above the horizontal. Find V and θ. [5]

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Q12

4 A small ball is projected with speed $25\ \text{m s}^{-1}$ at an angle of 30° above the horizontal from a point O on horizontal ground. At time t s after projection the horizontal and vertically upwards displacements of the ball from O are x m and y m respectively.

(i) Express x and y in terms of t and hence find the equation of the trajectory of the ball. [4]

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(ii) Find x for the position of the ball when its path makes an angle of 15° below the horizontal. [4]

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Q13

1 A small ball is projected from a point O on horizontal ground at an angle of 30° above the horizontal. At time t s after projection the vertically upwards displacement of the ball from O is $(14t - kt^2)$ m, where k is a constant.

(i) State the value of k. [1]

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(ii) Show that the initial speed of the ball is $28\ \text{m s}^{-1}$. [2]

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(iii) Find the horizontal displacement of the ball from O when $t = 3$. [2]

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Q14

6 A particle is projected with speed $15\,\text{m s}^{-1}$ at an angle of $\theta°$ above the horizontal. At the instant 4 s after projection the speed of the particle is $30\,\text{m s}^{-1}$.

(i) Find θ. [4]

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(ii) Show that at the instant 4 s after projection the particle is 33.75 m below the level of the point of projection and find the direction of motion at this instant. [4]

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2 A particle is projected from a point on horizontal ground with speed $15\,\text{m}\,\text{s}^{-1}$ at an angle of $\theta°$ above the horizontal. The particle strikes the ground 2 s after projection.

(i) Find θ. [2]

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(ii) Calculate the time after projection at which the direction of motion of the particle is 20° below the horizontal. [4]

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4 A small ball B is projected with speed $30\,\text{m s}^{-1}$ at an angle of 60° above the horizontal from a point O. At time t s after projection the horizontal and vertically upwards displacements of B from O are x m and y m respectively.

(i) Express x and y in terms of t and hence find the equation of the trajectory of the ball. [4]

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(ii) Find the value of x for which OB makes an angle of 45° above the horizontal. [3]

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Q17

2 A small ball is projected from a point O on horizontal ground at an angle of 30° above the horizontal. At time t s after projection the horizontal and vertically upwards displacements of the ball from O are x m and y m respectively. It is given that $x = 40t$.

(i) Calculate the initial speed of the ball, and express y in terms of t. [3]

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(ii) Hence find the equation of the trajectory of the ball. [2]

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Q18

4 A particle is projected from a point O on horizontal ground with speed $V\,\text{m}\,\text{s}^{-1}$ at an angle of 60° above the horizontal. At the instant 3 s after projection the direction of motion of the particle is 30° below the horizontal.

(i) Find V. [3]

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(ii) Calculate the distance of the particle from O at the instant 3 s after projection. [3]

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Q19

2 An object is projected with speed $15\,\text{m}\,\text{s}^{-1}$ at an angle of 35° above the horizontal from a point on horizontal ground. Find the speed and direction of motion of the object at time 2 s after the instant of projection. [5]

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Q20

4 A particle P is projected from a point O on horizontal ground. At the instant t s after projection, the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of the trajectory of P is $y = 3x - 0.05x^2$.

(i) Find the angle of projection and the initial speed of P. [3]

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(ii) Find the coordinates of P at the instant when OP makes an angle of 45° with the horizontal. [2]

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(iii) For the instant when P is at its greatest height above the ground, calculate this height and the corresponding value of t. [4]

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Q21

1 A small ball B is projected from a point O on horizontal ground. The initial velocity of B has horizontal and vertically upwards components of $18\,\text{m s}^{-1}$ and $25\,\text{m s}^{-1}$ respectively. For the instant 4 s after projection, find the speed and direction of motion of B. [4]

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Q22

4 A small object is projected from a point O with speed V m s^{-1} at an angle of 45° above the horizontal. At time t s after projection, the horizontal and vertically upwards displacements of the object from O are x m and y m respectively.

(i) Express x and y in terms of t, and hence find the equation of the path. [4]

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The object passes through the point with coordinates (24, 18).

(ii) Find V. [2]

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(iii) The object passes through two points which are 22.5 m above the level of O. Find the values of x for these points. [3]

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Q23

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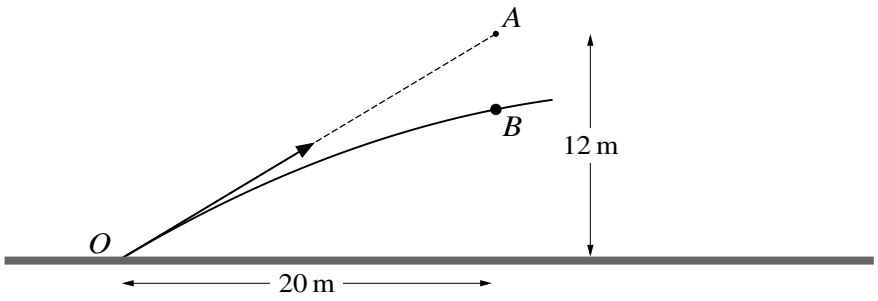

A small ball B is projected from a point O on horizontal ground towards a point A 12 m above the ground. 0.9 s after projection B has travelled a horizontal distance of 20 m and is vertically below A (see diagram).

(i) Find the angle and the speed of projection of B. [4]

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(ii) Calculate the distance AB when B is vertically below A. [2]

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Q24

4 A particle P is projected from a point O on horizontal ground with initial speed $20\,\text{m s}^{-1}$ and angle of projection 30°. At the instant t s after projection, the horizontal and vertically upwards displacements of P from O are x m and y m respectively.

(i) Express x and y in terms of t and hence find the equation of the trajectory of P. [4]

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P is at the same height above the ground at two points which are a horizontal distance apart of 15 m.

(ii) Calculate this height. [3]

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Q25

1 A small ball B is projected with speed $30\,\text{m s}^{-1}$ at an angle of $60°$ to the horizontal from a point on horizontal ground. Find the time after projection when the speed of B is $25\,\text{m s}^{-1}$ for the second time. [4]

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Q26

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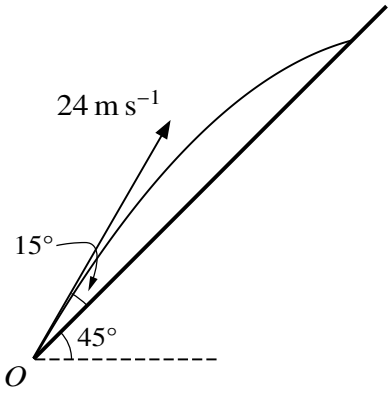

A small object is projected with speed 24 m s^{-1} from a point O at the foot of a plane inclined at 45° to the horizontal. The angle of projection of the object is 15° above a line of greatest slope of the plane (see diagram). At time t s after projection, the horizontal and vertically upwards displacements of the object from O are x m and y m respectively.

(i) Express x and y in terms of t, and hence find the value of t for the instant when the object strikes the plane. [4]

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(ii) Express the vertical height of the object above the plane in terms of t and hence find the greatest vertical height of the object above the plane. [5]

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Q27

1 A small ball B is projected with speed $38\,\text{m}\,\text{s}^{-1}$ at an angle of 30° to the horizontal from a point on horizontal ground. Find the speed of B when the path of B makes an angle of 20° above the horizontal. [3]

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Q28

4 A small object is projected horizontally with speed V m s^{-1} from a point O above horizontal ground. At time t s after projection, the horizontal and vertically upwards displacements of the object from O are x m and y m respectively.

(i) Express x and y in terms of t and hence show that the equation of the path of the object is $y = -\frac{5x^2}{V^2}$. [3]

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The object passes through points with coordinates $(a, -a)$ and $(a^2, -16a)$, where a is a positive constant.

(ii) Find the value of a. [3]

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(iii) Given that the object strikes the ground at the point where $x = 5a$, find the height of O above the ground . [2]

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Q29

1 A small ball is projected with speed $15\ \text{m s}^{-1}$ at an angle of 60° above the horizontal. Find the distance from the point of projection of the ball at the instant when it is travelling horizontally. [5]

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Q30

3 A particle P is projected with speed $20\,\text{m}\,\text{s}^{-1}$ at an angle of 60° below the horizontal, from a point O which is 30 m above horizontal ground.

(i) Calculate the time taken by P to reach the ground. [3]

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(ii) Calculate the speed and direction of motion of P immediately before it reaches the ground. [4]

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Q31

1 A particle is projected with speed $20\,\text{m}\,\text{s}^{-1}$ at an angle of 60° above the horizontal. Calculate the time after projection when the particle is descending at an angle of 40° below the horizontal. [4]

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Q32

4 A particle is projected from a point O on horizontal ground. The initial components of the velocity of the particle are $10\ \text{m s}^{-1}$ horizontally and $15\ \text{m s}^{-1}$ vertically. At time t s after projection, the horizontal and vertically upwards displacements of the particle from O are x m and y m respectively.

(i) Express x and y in terms of t, and hence find the equation of the trajectory of the particle. [4]

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The horizontal ground is at the top of a vertical cliff. The point O is at a distance d m from the edge of the cliff. The particle is projected towards the edge of the cliff and does not strike the ground before it passes over the edge of the cliff.

(ii) Show that d is less than 30. [2]

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(iii) Find the value of x when the particle is 14 m below the level of O. [2]

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Q33

7 A particle P is projected from a point O with speed V m s^{-1}. At time t s after projection the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of the trajectory of P is $y = 2x - \frac{25x^2}{V^2}$.

(i) Write down the value of $\tan\theta$, where θ is the angle of projection of P. [1]

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When $t = 4$, P passes through the point A where $x = y = a$.

(ii) Calculate V and a. [5]

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(iii) Find the direction of motion of P when it passes through A. [3]

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Q34

2 A small ball is projected from a point 1.5 m above horizontal ground. At a point 9 m above the ground the ball is travelling at 45° above the horizontal and its velocity is 4 m s^{-1}. Find the angle of projection of the ball. [4]

Craftear

Q35

4 A particle P is projected with speed $25\,\text{m s}^{-1}$ at an angle of $30°$ above the horizontal from a point O on horizontal ground. At time t s after projection the horizontal and vertically upwards displacements of P from O are x m and y m respectively.

(i) Express x and y in terms of t and hence show that the equation of the trajectory of P is

$$y = \frac{x}{\sqrt{3}} - \frac{4x^2}{375}.$$ [4]

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(ii) Find the horizontal distance between the two points at which P is 5 m above the ground. [3]

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Q36

7 A small ball B is projected from a point O which is h m above a horizontal plane. At time 2 s after projection B has speed 18 m s^{-1} and is moving in the direction 30° above the horizontal.

(i) Find the initial speed and the angle of projection of B. [4]

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B has speed 38 m s^{-1} immediately before it strikes the plane.

(ii) Calculate h. [2]

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B bounces when it strikes the plane, and leaves the plane with speed $20\,\text{m}\,\text{s}^{-1}$ but with its horizontal component of velocity unchanged.

(iii) Find the total time which elapses between the initial projection of B and the instant when it strikes the plane for the second time. [5]

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1 A particle is projected from a point on horizontal ground. At the instant 2 s after projection, the particle has travelled a horizontal distance of 30 m and is at its greatest height above the ground. Find the initial speed and the angle of projection of the particle. [5]

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3 A stone is thrown with speed $9\ \text{m s}^{-1}$ at an angle of 60° above the horizontal from a point on horizontal ground. Find the distance between the two points at which the path of the stone makes an angle of 45° with the horizontal. [5]

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Q39

1 A small ball is projected with speed $16\,\text{m}\,\text{s}^{-1}$ at an angle of 45° above the horizontal from a point on horizontal ground. Calculate the period of time, before the ball lands, for which the speed of the ball is less than $12\,\text{m}\,\text{s}^{-1}$. [4]

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5 A particle is projected at an angle of $\theta°$ below the horizontal from a point at the top of a vertical cliff 26 m high. The particle strikes horizontal ground at a distance 8 m from the foot of the cliff 2 s after the instant of projection. Find

(i) the speed of projection of the particle and the value of θ, [6]

(ii) the direction of motion of the particle immediately before it strikes the ground. [3]

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Q41

1 A small ball B is projected with speed $12\,\text{m}\,\text{s}^{-1}$ at an angle of 30° above the horizontal from a point O on horizontal ground. At the instant 0.8 s after projection, B is 0.5 m vertically above the top of a vertical post.

(i) Calculate the height of the top of the post above the ground. [3]

(ii) Show that B is at its greatest height 0.2 s before passing over the post. [2]

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[Turn over

3 The point O is 8 m above a horizontal plane. A particle P is projected from O. After projection, the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of the trajectory of P is

$$y = 2x - x^2.$$

(i) Find the value of x for the point where P strikes the plane. [2]

(ii) Find the angle and speed of projection of P. [3]

(iii) Calculate the speed of P immediately before it strikes the plane. [2]

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Q43

7 A particle P is projected with speed $35\ \text{m s}^{-1}$ from a point O on a horizontal plane. In the subsequent motion, the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of the trajectory of P is

$$y = kx - \frac{(1 + k^2)x^2}{245},$$

where k is a constant. P passes through the points $A\,(14,\ a)$ and $B\,(42,\ 2a)$, where a is a constant.

(i) Calculate the two possible values of k and hence show that the larger of the two possible angles of projection is 63.435°, correct to 3 decimal places. [5]

For the larger angle of projection, calculate

(ii) the time after projection when P passes through A, [2]

(iii) the speed and direction of motion of P when it passes through B. [4]

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[Turn over

Q44

1 A stone S is thrown horizontally from the top T of a high tower. At the instant 1.6 s after S is thrown, the line ST makes an angle of 30° below the horizontal. Find the speed with which S is thrown. [3]

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Q45

4 A particle P is projected with speed $20\,\text{m s}^{-1}$ at an angle of $30°$ above the horizontal from a point O on horizontal ground. P subsequently bounces when it first strikes the ground at the point A.

(i) Find the time after projection when P first strikes the ground, and the distance OA. [3]

When P bounces at A the horizontal component of the velocity of P is unchanged. The vertical component of velocity is $8\,\text{m s}^{-1}$ immediately after bouncing. P strikes the ground for the second time at B where it remains at rest.

(ii) Calculate the first and last times after projection at which the speed of P is $18\,\text{m s}^{-1}$. [5]

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Q1

Question	Answer	Marks	Guidance
7(a)	At greatest height $0 = 100\sin\theta - gt$	**M1**	
	$t = 8$	**A1**	
	Therefore times at height H are $t = 3$ (and $t = 13$)	**B1**	
	Substitute into $H = 100\sin\theta t - \frac{1}{2}gt^2$	**M1**	
	$H = 195$	**A1**	
	Alternative method to question **7(a)**		
	$\uparrow H = 100\sin\theta t - \frac{1}{2}gt^2$	**M1**	
	And $H = 100\sin\theta(t+10) - \frac{1}{2}g(t+10)^2$	**A1**	
	Subtract: $1000\sin\theta = \frac{1}{2}g(20t+100)$	**M1**	
	$t = 3$	**B1**	
	$H = 195$	**A1**	
7(a)	Alternative method to question **7(a)**		
	$\uparrow H = 100\sin\theta t - \frac{1}{2}gt^2$	**B1**	
	Difference between roots = $\frac{\sqrt{(100\sin\theta)^2 - 2gH}}{\frac{1}{2}g}$	**M1 A1**	
	Equate to 10 and rearrange to find H	**M1**	
	$H = 195$	**A1**	
		5	
7(b)	Time to required point = 15 s	**B1**	
	$\uparrow v = 100\sin\theta - 10\times 15 (= -70)$ $\rightarrow v = 100\cos\theta = 60$	**B1**	Both components.
	Magnitude = 92.2	**B1**	
	Angle below horizontal = $\tan^{-1}(70/60) = 49.4°$	**B1**	
		4	

Q2

Question	Answer	Marks	Guidance
7(a)	$y = 0$ in trajectory equation: $R\tan\theta - g\frac{R^2}{2u^2(\cos\theta)^2} = 0$	**M1**	
	$(R =)\frac{2u^2\sin\theta\cos\theta}{g}$ only	**A1**	Any equivalent single term expression, for example: $\frac{u^2\sin 2\theta}{g}$, $\frac{2u^2\tan\theta}{g\sec^2\theta}$, at least one intermediate line of working, not just quoting a result. SC B1 using SUVAT.
		2	
7(b)	$x = their\frac{u^2\sin\theta\cos\theta}{g}$ and substitute in trajectory equation.	**M1**	Or use SUVAT.
	$H = \frac{u^2(\sin\theta)^2}{2g}$	**A1**	Single term.
		2	
7(c)	Use $R = \frac{4H}{\sqrt{3}}$ and simplify: $\tan\theta = \sqrt{3}$, $\theta = 60°$	**B1**	AG
		1	

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7(d)	$\frac{dy}{dx} = \tan\theta - \frac{gx}{u^2(\cos\theta)^2}$	**M1**	Differentiate with respect to x.
	$\tan\theta - \frac{x}{4(\cos\theta)^2} = \pm 1$ used	**M1**	Use $\frac{dy}{dx} = \pm 1$ as limiting case.
	$x = \sqrt{3}+1$, $x = \sqrt{3}-1$	**A1**	
	$\sqrt{3}-1 < x < \sqrt{3}+1$	**A1**	Strict inequality, exact values.
	Alternative method for question 7(d)		
	$y = \sqrt{3}x - \frac{1}{2}x^2$, $\frac{dy}{dx} = \sqrt{3} - x$	**M1**	Differentiate with respect to x.
	$\frac{dy}{dx} = \pm 1$ used	**M1**	Use $\frac{dy}{dx} = \pm 1$
	$x = \sqrt{3}+1$, $x = \sqrt{3}-1$	**A1**	
	$\sqrt{3}-1 < x < \sqrt{3}+1$	**A1**	Strict inequality, exact values.

7(d)	**Alternative method for question 7(d)**		
	When moving at 45° to horizontal, $v_x = \pm v_y$	**M1**	Used, both cases considered.
	$v_x = \sqrt{40}\cos\theta$, $v_y = \sqrt{40}\sin\theta - 10t$ $t = \frac{1}{10}\left(\sqrt{30}-\sqrt{10}\right)$, $t = \frac{1}{10}\left(\sqrt{30}+\sqrt{10}\right)$	**M1**	as limiting case.
	$x = \sqrt{3}+1$, $x = \sqrt{3}-1$	**A1**	
	$\sqrt{3}-1 < x < \sqrt{3}+1$	**A1**	Strict inequality, exact values.
		4	

Q3

Question	Answer	Marks	Guidance
5	At A: $\uparrow u\sin\theta - 8g$ $\quad\rightarrow u\cos\theta$	**M1**	Both.
	$\tan\alpha = \frac{u\sin\theta - 8g}{u\cos\theta}$	**A1**	
	At B: $\uparrow u\sin\theta - 32g$ $\quad\rightarrow u\cos\theta$	**M1**	Both.
	$\tan\beta = \frac{u\sin\theta - 32g}{u\cos\theta}$	**A1**	
	$\frac{u\sin\theta - 8g}{u\cos\theta} \times \frac{u\sin\theta - 32g}{u\cos\theta} = -1$	**B1**	Perpendicular directions, so $\tan\alpha \times \tan\beta = -1$.
	$u^2 - 320u + 25600 = 0$	**M1**	Simplify to a quadratic in u.
	$u = 160$	**A1**	
		7	

Q4

Question	Answer	Marks	Guidance
1(a)	Velocity: $\rightarrow u\cos\alpha$	**B1**	
	$\uparrow u\sin\alpha - gT$	**B1**	Allow 10 for g. Must be T.
		2	
1(b)	$\frac{u\cos\alpha}{u\sin\alpha - gT} = -\frac{\sin\alpha}{\cos\alpha}$ oe	**M1 FT**	Allow missing minus sign on RHS for M1. FT from **(a)**.
	$T = \frac{u}{g\sin\alpha}$	**A1**	
		2	
1(c)	$\sin\alpha < 1$ giving $T > \frac{u}{g}$	**B1**	AG
		1	

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Q5

Question	Answer	Marks
1	For greatest height, $T = \frac{u}{2g}$	B1
	At $t = \frac{2T}{3}, \uparrow v_v = \frac{u}{2} - \frac{2Tg}{3} = \frac{u}{6}$	M1
	$\rightarrow v_h = \frac{u\sqrt{3}}{2}$	A1
	Speed = $\sqrt{v_v^2 + v_h^2} = \sqrt{\frac{u^2}{36} + \frac{3u^2}{4}}$	M1
	$= \frac{\sqrt{7}}{3}u$	A1
		5

Q6

Question	Answer	Marks
6(a)	Greatest height = $\frac{(u\sin\theta)^2}{2g}$	M1A1
	At ¾ greatest height, $\rightarrow v\cos\alpha = u\cos\theta$	M1
	At ¾ greatest height, $\uparrow (v\sin\alpha)^2 = (u\sin\theta)^2 - 2g.\frac{3}{4}\frac{(u\sin\theta)^2}{2g}$	M1
	$v\sin\alpha = \frac{1}{2}u\sin\theta$	A1
	So $\tan\alpha = \frac{1}{2}\tan\theta$ **AG**	A1
	–	6
6(b)	$\tan\alpha = \frac{1}{2}.\frac{4}{3} = \frac{2}{3}$ $\uparrow \frac{1}{2}u\sin\theta = u\sin\theta - gt$	M1
	$t = \frac{2u}{5g}$	A1
	$\rightarrow t = d / u\cos\theta$	M1
	$= \frac{6u^2}{25g}$	A1
		4

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Q7

Question	Answer	Marks	Guidance
5(a)	$\rightarrow x = u\cos\alpha t$ $\uparrow y = u\sin\alpha t - \frac{1}{2}gt^2$	**B1**	Both
	Eliminate t: $y = u\sin\alpha.\frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$	**M1**	Eliminate
	$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha$	**A1**	AG
		3	
5(b)	Greatest height $= \frac{(u\sin\alpha)^2}{2g} = \frac{u^2}{4g}$	**M1 A1**	Accept alternative methods, for example differentiate expression in **(a)** and equate to 0.
	$t = u\sin 45/g$ so $d = u\cos 45.u\sin 45/g = \frac{u^2}{2g}$	**A1**	AG
		3	
5(c)	Use greatest height displacements in trajectory equation $\frac{u^2}{4g} = \frac{u^2}{2g}\tan\alpha - \frac{gu^4}{2u^2 4g^2}\sec^2\alpha$	**M1**	Use equation of trajectory (substitute coordinates of Q
	$u^2 = 2u^2\tan\alpha - \frac{u^2}{2}(1+\tan^2\alpha)$	**M1**	Use of $\sec^2\alpha = (1+\tan^2\alpha)$
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$	**M1**	Obtain a three-term quadratic in $\tan\alpha$
	$\tan\alpha = 1,\ 3$ so $\alpha = 71.6°$	**A1**	Both solutions needed
		4	

Q8

Question	Answer	Marks	Guidance
5(a)	Quote trajectory equation from MF19 and use $\cos\theta = 1/\sec\theta$ $y = x\tan\theta - \frac{gx^2}{2u^2}(1+\ \tan^2\theta)$	**B1**	Must include step with $\sec^2\theta$ Allow derived from first principles AG
		1	
5(b)	$16 = 20 - \frac{10\times100}{2u^2}(1+4)$	**M1**	Substitute into result **(a)**
	$u^2 = 625,\quad (u = 25)$	**A1**	
	Use equation again: $30 = 18\tan\theta - \frac{10\times324}{2\times625}(1+(\tan\theta)^2)$	**M1**	
	$2.592(\tan\theta)^2) - 18\tan\theta + 32.592 = 0$	**A1**	3 term quadratic. Alternatives include: $54t^2 - 375t + 679 = 0$, $324t^2 - 2250t + 4074 = 0$
	Discriminant $= 324 - 4\times2.592\times32.592 = -13.91$	**M1**	Discriminant for alternatives: −6039 and −217404
	As this is less than 0, no real solutions for θ	**A1**	CWO
		6	

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Q9

Question	Answer	Marks	Guidance
1	$x' = 24\cos 30$ ($= 12\sqrt{3}$)	**B1**	Use horizontal motion
	$y' = 24\sin 30 - 4g$ ($= -28$)	**B1**	Use vertical motion
	$V^2 = (24\cos 30)^2 + (24\sin 30 - 4g)^2 = (12\sqrt{3})^2 + (-28)^2$ OR $\tan\alpha = (24\sin 30 - 4g)/(24\cos 30) = -28/(12\sqrt{3})^2$	**M1**	Where V is the required speed and α is the angle below the horizontal
	V = 34.9 m s^{-1}	**A1**	
	α = 53.4° below the horizontal	**A1**	
		5	

Q10

Question	Answer	Marks	Guidance
3(i)	$x = 4t$ and $y = 6t - 5t^2$	**M1**	Use horizontal and vertical motion and attempt to eliminate t
	$y\ [= 6x/4 - 5(x/4)^2] = 1.5x - 5x^2/16$ or $1.5x - 0.3125x^2$	**A1**	
		2	
3(ii)	$\theta = 1.5$	**M1**	Use the trajectory equation from the formula sheet
	$\theta = 56.3°$	**A1**	
	$V^2\cos^2 56.3 = 16$	**M1**	Again use the trajectory equation
	V = 7.21 m s^{-1}	**A1**	
	OR		
	$V\cos\theta = 4$ and $V\sin\theta = 6$	**M1**	Initial horizontal and vertical velocities
	$V^2\cos^2\theta + V^2\sin^2\theta = 4^2 + 6^2$ OR $\tan\theta = 6/4$	**M1**	Use Pythagoras's theorem or trigonometry of a right angled triangle
	V = 7.21 m s^{-1}	**A1**	
	$\theta = 56.3°$	**A1**	
		4	

Q11

Question	Answer	Marks	Guidance
2	$V\cos\theta = 16\cos 30 \left(= 8\sqrt{3} = 13.856...\right)$	**B1**	Use horizontal motion
	$V\sin\theta = 16\sin 30 + 4g \left(= 48\right)$	**B1**	Use vertical motion
	$V^2 = (16\cos 30)^2 + (16\sin 30 + 4g)^2$ **OR** $\tan\theta = \dfrac{(16\sin 30 + 4g)}{16\cos 30}$	**M1**	Use Pythagoras's theorem or trigonometry of a right angled triangle
	$V = 50(.0)$	**A1**	
	$\theta = 73.9°$	**A1**	
		5	

Q12

Question	Answer	Marks	Guidance
4(i)	$x = 25\cos 30t$	**B1**	Use horizontal motion
	$y = 25\sin 30t - \dfrac{gt^2}{2}$	**B1**	Use vertical motion
	$y = 25\sin 30\left(\dfrac{x}{25\cos 30}\right) - \dfrac{g\left(\dfrac{x}{25\cos 30}\right)^2}{2}$	**M1**	Eliminate t
	$y = \dfrac{x}{\sqrt{3}} - \dfrac{4x^2}{375}$ or $y = 0.577x - 0.0107x^2$	**A1**	
		4	

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4(ii)	$\frac{dy}{dx} = \frac{1}{\sqrt{3}} - \frac{8x}{375}$ or $\frac{dy}{dx} = 0.577 - 0.0214x$	**M1A1**	Differentiate the equation from part **(i)** to find the gradient
	$-\tan 15 = \frac{1}{\sqrt{3}} - \frac{8x}{375}$ or $-\tan 15 = 0.577 - 0.0214x$	**M1**	Attempt to solve
	$x = 39.6$ or $x = 39.5$	**A1**	
		4	
	Alternative method for question 4(ii)		
	$\tan 15 = \frac{v_y}{v_x} = \frac{v_y}{12.5\sqrt{3}}$	**M1**	
	$v_y = 12.5\sqrt{3}\tan 15 \left(= 5.8\right)$ downwards	**A1**	
	$-5.8 = 12.5 - 10t$ leading to $t = 1.83$	**M1**	Vertical motion using $v = u + at$
	$X = 1.83 \times \frac{25\sqrt{3}}{2} = 39.6$	**A1**	
		4	

Q13

Question	Answer	Marks	Guidance
1(i)	$k = \frac{g}{2} = 5$	**B1**	Use the trajectory equation from the formula sheet
		1	
1(ii)	$V\sin 30 = 14$	**M1**	Use the trajectory equation from the formula sheet
	$V = 28$ ms^{-1} **AG**	**A1**	
		2	
1(iii)	$x = 28\cos 30 \times 3$	**M1**	Use horizontal motion. Allow *their* V for M1
	$x = 72.7$ m	**A1**	
		2	

Q14

Question	Answer	Marks	Guidance
6(i)	$15\cos\theta = v_H$ and $15\sin\theta - 4g = v_V$	**B1**	Use horizontal and vertical motion
	$(15\cos\theta)^2 + (15\sin\theta - 4g)^2 = 30^2$	**M1**	Use Pythagoras's theorem
	$[225 - 1200\sin\theta + 1600 = 900]$	**M1**	Attempt to solve for θ
	$\theta = 50.4°$	**A1**	
		4	

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6(i)	**Alternative Method**		
	$h = (15\sin\theta)\times 4 - \frac{g(4)^2}{2}$	**B1**	
	$\frac{m(15)^2}{2} = \frac{m(30)^2}{2} + mgh$	**M1**	Allow h not replaced
		M1	Attempt to eliminate h and attempt to solve for θ
	$\theta = 50.4°$	**A1**	
		4	
6(ii)	$s = 15\sin 50.4 \times 4 - \frac{1}{2}\times g \times 4^2$	**M1**	Use vertical motion. Allow *their* θ for first M1
	$s = 33.75$ m **AG**	**A1**	
	$\cos\alpha = \frac{15\cos 50.4}{30}$	**M1**	Use trigonometry of a right angled triangle
	$\alpha = 71.4°$ below the horizontal	**A1**	
		4	If g = 9.8 or 9.81 used then M1A0M1A0

Q15

Question	Answer	Marks	Guidance
2(i)	$-15\sin\theta = 15\sin\theta - 2g$	**M1**	Use $v = u + at$ vertically
	$(\theta =)\ 41.8$	**A1**	
		2	
2(ii)	Vertically: $\frac{v}{15\cos\theta} = \pm\tan 20$	**M1**	v = vertical velocity
	$v = (\pm)4.07$	**A1**	
	$-4.07 = 15\sin 41.8 - gt$	**M1**	Use $v = u + at$ vertically
	$(t =)\ 1.41$ s	**A1**	
		4	

Q16

Question	Answer	Marks	Guidance
4(i)	$x = 30\cos 60t$	**B1**	Use horizontal motion
	$y = 30\sin 60t - \frac{gt^2}{2}$	**B1**	Use $s = ut + \frac{gt^2}{2}$ vertically
	$y = \frac{30\sin 60x}{30\cos 60} - \frac{5x^2}{(30\cos 60)^2}$	**M1**	Attempt to eliminate t
	$y = 1.73x - 0.0222x^2$ or $y = \sqrt{3}x - \frac{x^2}{45}$	**A1**	
		4	
4(ii)	$x = y$ or $\tan 45 = \frac{y}{x}$	**M1**	
	$1 = 1.73 - 0.0222x$ or $1 = \sqrt{3} - \frac{x}{45}$	**M1**	x common to all three terms
	$x = 32.9$	**A1**	
		3	

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Q17

Question	Answer	Marks	Guidance
2(i)	$V\cos 30 = 40$	**M1**	Note V is the velocity of projection
	$V = 46.2\ \text{m}\,s^{-1}$	**A1**	Allow $\frac{80}{\sqrt{3}}$ or $\frac{80\sqrt{3}}{3}$
	$y = 23.1t - 5t^2$	**B1FT**	Use $s = ut + \frac{at^2}{2}$ vertically. FT candidates half V but not $V = 40$ used
		3	
2(ii)	$y = \frac{23.1x}{40} - \frac{5x^2}{1600}$	**M1**	Attempt to eliminate t by substituting $t = \frac{x}{40}$ into answer to **part (i)**
	$y = 0.577x - \frac{x^2}{320}$ or $y = 0.577x - 0.003125x^2$	**A1**	
		2	

Q18

Question	Answer	Marks	Guidance
4(i)	Velocity component vertically = $\pm$ ($V\sin 60 - 3g$)	**B1**	Use $v = u + at$
	$\tan 30 = \frac{30 - V\sin 60}{V\cos 60}$	**M1**	Use trigonometry of a right angled triangle
	$V = 15\sqrt{3} = 26(.0)\ \text{m s}^{-1}$	**A1**	
		3	
4(ii)	$y = 26\sin 60 \times 3 - \frac{g \times 3^2}{2}$	**B1FT**	Use $s = ut + \frac{at^2}{2}$ vertically. Their V from **part (i)**
	$D^2 = (26\sin 60 \times 3 - g \times 3^2)^2 + (26\cos 60 \times 3)^2$	**M1**	Use Pythagoras's Theorem
	$D = 45(.0)\ \text{m}$	**A1**	
		3	

Q19

Question	Answer	Marks	Guidance
2	u = 15cos35 (= 12.287)	**B1**	Use horizontal motion
	v = 15sin35 – 2g (= –11.396)	**B1**	Use vertical motion
	$V = \sqrt{(12.287^2 + 11.396^2)}$ OR $\tan\theta = \pm 11.396 / 12.287$	**M1**	
	$V = 16.8\ \text{m}\,s^{-1}$	**A1**	
	$\theta = 42.8°$ below the horizontal	**A1**	
		5	

Q20

Question	Answer	Marks	Guidance
4(i)	$\theta = 3)\ \theta = 71.6°$	**B1**	Use the formula sheet for the trajectory equation
	$0.05 = g / (2V^2 cos^2 71.6)$	**M1**	
	$V = 10\sqrt{10} = 31.6\ \text{m}\,s^{-1}$	**A1**	
		3	
4(ii)	$x = 3x - 0.05x^2$	**M1**	Use y = x
	x = 40 and y = 40	**A1**	
		2	

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4(iii)	y / dx = 3 – 0.1x = 0	**M1**	Use the fact that the gradient is zero at the highest point
	x = 30, y = (3 × 30 – 0.05 × 30^2) = 45	**A1**	
	30 = (31.6cos71.6)t	**M1**	Use horizontal motion
	t = 3.01	**A1**	t = 3 if exact arithmetic used
		4	

Q21

Question	Answer	Marks	Guidance
1	Vertical component of velocity = 25 – 4g	**M1**	Use $v = u + at$
	$v^2 = 18^2 + (25-4g)^2$ **or** $\tan\theta = \dfrac{(25-4g)}{18}$	**M1**	
	$v = 23.4$ ms^{-1}	**A1**	
	$\theta = 39.8°$ below the horizontal	**A1**	
		4	

Q22

Question	Answer	Marks	Guidance
4(i)	$x = (V\cos 45)t$	**B1**	Use horizontal motion
	$y = (V\sin 45)t - \dfrac{gt^2}{2}$	**B1**	Use $s = ut + \frac{1}{2}gt^2$ vertically
	$y = \dfrac{(V\sin 45)x}{(V\cos 45)} - \dfrac{1}{2}g\left(\dfrac{x}{V\cos 45}\right)^2$	**M1**	Attempt to eliminate t
	$y = x - \dfrac{10x^2}{V^2}$	**A1**	
		4	
4(ii)	$18 = 24 - \dfrac{10\times 24^2}{V^2}$	**M1**	Substitutes x = 18, y = 24 in part **(i)** equation
	V = 31(.0)	**A1**	
		2	
4(iii)	$22.5 = x - \dfrac{10x^2}{960}$	**M1**	Put y = 22.5 in part **(i)**
	$x^2 - 96x + 2160 = 0$	**M1**	Attempt to solve a quadratic equation
	x = 36, 60	**A1**	
		3	

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Q23

Question	Answer	Marks	Guidance
1(i)	$\tan\theta = 12/20$	**M1**	θ is the angle of projection
	$\theta\ (= 30.96) = 31(.0)°$	**A1**	
	$V\cos 30.96 = \frac{20}{0.9}$	**M1**	Use horizontal motion. Allow their θ for the M mark.
	$V = 25.9\ \text{m}\,s^{-1}$	**A1**	
	Total:	**4**	
1(ii)	$H = 25.9\sin 31 \times 0.9 - g \times \frac{0.9^2}{2}\ (= 7.948)$	**M1**	Use $s = ut + \frac{1}{2}at^2$ vertically. H is the height above the ground. Allow their V and θ for the M mark.
	AB (= 12 – 7.95) = 4.05 m	**A1**	Allow AB = 4.06
	Total:	**2**	

Q24

Question	Answer	Marks	Guidance
4(i)	$x = (20\cos 30)t$ or $10\sqrt{3}\ t$	**B1**	Use horizontal motion
	$y = (20\sin 30)t - \frac{1}{2}g t^2$ or $10t - 5t^2$	**B1**	Use vertical motion
	$y = (20\sin 30)[x/(20\cos 30)] - 5[x/(20\cos 30)]^2$	**M1**	Attempt to eliminate t
	$y = x/\sqrt{3} - x^2/60$ or $0.577x - 0.0167x^2$	**A1**	
	Total:	**4**	
4(ii)	$x/\sqrt{3} - x^2/60 = (x+15)/\sqrt{3} - (x+15)/60$	**M1**	Simplifies to $0 = 15/\sqrt{3} - (30x+225)/60$
	$x = 9.821$	**A1**	
	y = 4.06(25) m	**A1**	
	Total:	**3**	
	Alternative method		
	$0.577x - 0.0167x^2 = 0.577(x+15) - 0.0167(x+15)^2$	**M1**	
	$x = 9.775$	**A1**	
	$y = 4.044$	**A1**	
	Total:	**3**	

Q25

Question	Answer	Marks	Guidance
1	$v^2 = 25^2 - (30\cos 60)^2$	**M1**	v = vertical velocity at the required point
	v = (±) 20	**A1**	
	–20 = 30sin60 – gt	**M1**	Use v = u + at vertically
	t = 4.6(0) s	**A1**	
		4	

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Q26

Question	Answer	Marks	Guidance
7(i)	$x = (24\cos60)t$	**B1**	Use horizontal motion
	$y = (24\sin60)t - gt^2/2$	**B1**	Use vertical motion
	$(24\cos60)t = (24\sin60)t - gt^2/2$	**M1**	Recognise that $x = y$
	$t = 1.76$	**A1**	
		4	
7(ii)	$h = (24\sin60)t - gt^2/2 - (24\cos60)t$	**B1**	
		M1	Attempt to differentiate
	$dh/dt = 24(\sin60 - \cos60) - gt$	**A1**	
	$24(\sin60 - \cos60) - gt = 0$, $t = 0.878(46..)$	**M1**	Equate $dh/dt = 0$ to find t
	$h = 3.86$ m	**A1**	
		5	

Q27

Question	Answer	Marks	Guidance
1		**M1**	Equate initial horizontal velocity to final horizontal velocity
	$v\cos20 = 38\cos30$	**A1**	
	$v = 35(.0)$ m s^{-1}	**A1**	
		3	

Q28

Question	Answer	Marks	Guidance
4(i)	$x = Vt$ **or** $y = -\frac{1}{2}gt^2$	**B1**	Use horizontal motion or vertical motion
		M1	Attempt to eliminate t
	$y = -5x^2/V^2$ **AG**	**A1**	
		3	
4(ii)	$-a = -5a^2/V^2$ **or** $-16a = -5(a^2)^2/V^2$	**B1**	
		M1	Attempt to eliminate V
	$a = 4$	**A1**	
		3	
4(iii)	$y = -5(5\times4)^2/(5\times4)$ $(= -100)$	**M1**	Use the value of a found in part ii
	Height = 100 m	**A1**	
		2	

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Q29

Question	Answer	Marks	Guidance
1	$0 = 15\sin60 - gt$ $(t = 3\sqrt{3}/4 = 1.30)$	**M1**	Uses $v = u + at$ vertically
	$x = 15\cos60$ x 1.3 $(= 9.7428..)$	**A1**	
	$y = (15\sin60)^2/(2g)$ $(= 8.4375..)$	**B1**	Uses $v^2 = u^2 + 2as$ vertically
	$D = \sqrt{(9.74^2 + 8.44^2)}$	**M1**	Applies Pythagoras's theorem
	$D = 12.9$ m	**A1**	
	Total:	**5**	

Q30

Question	Answer	Marks	Guidance
3(i)	$30 = (20\sin 60)t + \frac{1}{2}gt^2$	**M1**	Uses $s = ut + \frac{1}{2}at^2$ vertically
	$5t^2 + 10\sqrt{3}\ t - 30 = 0$	**M1**	Sets up a quadratic equation and attempts to solve it
	$t = 1.27$	**A1**	
	Total:	**3**	
3(ii)	$v^2 = (20\sin 60)^2 + 2g$ x 30 (hence v = 30)	**B1**	Uses $v^2 = u^2 + 2as$ vertically
	$V = \sqrt{30^2 + (20\cos 60)^2}$ or $\tan\theta = 30/(20\cos 60)$	**M1**	
	$V = 31.6$ ms^{-1}	**A1**	
	$\theta = 71.6°$ with the horizontal	**A1**	Or 18.4° with the downward vertical
	Total:	**4**	

Q31

Question	Answer	Marks	Guidance
1	$\tan 40 = v / 20\cos 60$	**M1**	
	$v = 10\tan 40$ (= 8.3909...)	**A1**	
	$-10\tan 40 = 20\sin 60 - gt$	**M1**	Uses $v = u + at$ vertically
	$t = 2.57$ s	**A1**	
	Total:	**4**	

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Q32

4(i)	$x = 10t$ or $y = gt^2 / 2$	**B1**	
	$y = 15x / 10 - g(x / 10)^2 / 2$	**M1A1**	Attempts to eliminate t
	$y = 1.5x - 0.05x^2$	**A1**	
	Total:	**4**	
4(ii)	$0 = 1.5x - 0.05x^2$	**M1**	Substitute $y = 0$ into the trajectory equation
	$x = 30$	**A1**	
	Total:	**2**	
4(iii)	$-14 = 1.5x - 0.05x^2$	**M1**	Sets up a quadratic equation and attempts to solve it
	$x = 37.5$	**A1**	
	Total:	**2**	

Q33

7(i)	$\tan\theta = 2$	**B1**	Note $\theta = 63.4349..°$
	Total:	**1**	
7(ii)	*EITHER:* $a = 2a - 25a^2 / V^2$ $(25a = V^2)$	**(B1**	Substitutes $x = y = a$ into the trajectory equation
	$a = V\cos 63.4349.. \times 4$	**B1**	Horizontal motion
	$V^2 = 25 \times 4 \times V\cos 63.4349..$	**M1**	Attempts to eliminate a
	$V = 44.7(213..)$ or $20\sqrt{5}$	**A1**	
	$a = 80$	**A1)**	
	OR: $a = V\sin 63.4349.. \times 4 - g4^2/2$	**(B1**	Uses $s = ut + at^2/2$ vertically
	$a = V\cos 63.4349.. \times 4$	**B1**	Horizontal motion
	$V\sin 63.4349.. \times 4 - g4^2/2 = V\cos 63.4349.. \times 4$	**M1**	Attempts to solve the 2 equations
	$V = 44.7(213..)$or $20\sqrt{5}$	**A1**	
	$a = 80$	**A1)**	
	Total:	**5**	
7(iii)	$v_v = 44.7213..\sin 63.4349.. - 4g\ (= 0)$	**M1**	v_v = vertical component of the velocity
	$\alpha = \tan^{-1} +/- 0 / (44.7213..\cos 63.4349..)$	**M1**	$\tan\alpha = v_v / v_h$ where v_h = horizontal velocity
	$\alpha = 0°$	**A1**	
	Total:	**3**	

Q34

Question	Answer	Marks	Guidance
2	$V\cos\theta = 4\cos 45$	**B1**	Using horizontal motion with V = velocity of projection and θ = angle of projection.
	$(4\sin 45)^2 = (V\sin\theta)^2 - 2g(9-1.5)$ (leads to $V\sin\theta = \sqrt{158}$)	**M1**	Uses $v^2 = u^2 + 2as$ vertically.
	$\tan\theta = \sqrt{158}/(4\cos 45)$	**M1**	Uses trigonometry.
	$\theta = 77.3°$	**A1**	
		4	

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Q35

Question	Answer	Marks	Guidance
4(i)	$x = (25\cos30)t$	**B1**	Horizontal motion.
	$y = (25\sin30)t - gt^2/2$	**B1**	Vertical motion.
	$y = (25\sin30)x / (25\cos30) - 5[x/(25\cos30)]^2$	**M1**	Attempts to eliminate t.
	$y = \frac{x}{\sqrt{3}} - \frac{4x^2}{375}$	**A1**	AG
		4	
4(ii)	$5 = x/\sqrt{3} - 4x^2/375$ (leads to $4x^2 - 216.5x + 1875 = 0$)	**M1**	Substitutes $y = 5$ into the trajectory equation.
	$x = 43.3, 10.8$	**A1**	Solves the quadratic equation.
	Distance = 43.3 – 10.8 = 32.5 m	**A1**	
		3	

Q36

Question	Answer	Marks	Guidance
7(i)	$U_H = 18\cos30$ and $U_V = 18\sin30 + 2g(=29)$	**B1**	
	$U = \sqrt{[(18\cos30)^2 + 29^2]}$ or $\tan\theta = 29/(18\cos30)$	**M1**	Uses Pythagoras's Theorem and trigonometry.
	$U = 32.9(24..)\ \text{m s}^{-1}$	**A1**	
	$\theta = 61.7°$	**A1**	
		4	
7(ii)	$v^2 = 38^2 - (18\cos30)^2 = (+/-29)^2 + 2gh$	**M1**	Uses 2 ways to find v, the vertical velocity at the ground and equates.
	$h = 18$	**A1**	
	OR $mgh + m \times 32.924^2/2 = \text{m} \times 38^2/2$	**M1**	
	$h = 18$	**A1**	
		2	
7(iii)	$-\sqrt{[(38^2 - (18\cos30)^2]} = 29 - gt$	**M1**	Uses $v = u + at$ for first part of flight.
	$t = 6.36(6)$	**A1**	
	$v = \sqrt{[20^2 - (18\cos30)^2]} = 12.5(3)$ $-12.5(3) = 12.5(3) - gt'$	**M1**	Uses $v = u + at$ for second part of flight.
	$t' = 2.50(6)$	**A1**	
	$T\ (= 6.366 + 2.506) = 8.87$	**A1**	
		5	

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Q37

1	$V\sin\theta = 2g\ (= 20)$	**B1**		Using vertical motion to greatest height
	$V\cos\theta = 30/2\ (= 15)$	**B1**		Using horizontal motion
	$V^2 = 15^2 + 20^2$ or $\tan\theta = 20/15$	**M1**		Using Pythagoras or trigonometry
	$V = 25\ \text{m}s^{-1}$	**A1**		
	$\theta = 53.1°$	**A1**	5	

Q38

3	$V_h = 9\cos60$	**B1**		
	$V_v = (\pm)4.5$	**B1**		Or $9\cos60$
	$-4.5 = 4.5 - g$	**M1**		
	$t = 0.9$	**A1**		
	Distance = 4.05	**A1**	5	From $0.9 \times 9\cos60$

Q39

1	$v^2 = 12^2 - (16\cos45)^2$	**M1**		
	$v = 4$	**A1**		
	$-4 = 4 - gt$	**M1**		
	$t = 0.8\,\text{s}$	**A1**	4	

Q40

5 (i)	$v\cos\theta = 8/2$	**B1**		
	$-26 = -2v\sin\theta - g\,2^2/2$	**M1**		Accept with sign errors
	$v\sin\theta = 3$	**A1**		
	$v^2 = (+/-3)^2 + 4^2$ or $\tan\theta = 3/4$	**M1**		
	$v = 5\,\text{m}\,s^{-1}$	**A1**		
	$\theta = 36.9$	**A1**	6	
(ii)	$v_v = (+/-3) + 2g$	**M1**		
	$\tan\alpha = v_v/(8/2)$	**M1**		
	Angle = 80.1 with the horizontal or angle = 9.9 with the vertical	**A1**	3	

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Q41

1 (i)		**M1**		For attempting to find the height above the ground at the position of the post
	$s = (12\sin 30) \times 0.8 - g \times 0.8^2/2$	**A1**		1.6 m
	$H\ (= 1.6 - 0.5) = 1.1$ m	**A1**	3	
(ii)	$0 = 12\sin 30 - gt$	**M1**		$t = 0.6$
	Time $(= 0.8 - 0.6) = 0.2$ **AG**	**A1**	2	

Q42

3 (i)	$-8 = 2x - x^2$	**M1**		Sub y = −8 in the given equation with an attempt to solve
	$x = 4$	**A1**	2	
(ii)	$\theta = 63.4°$	**B1**		From $\tan^{-1} 2$
	$-g x^2/(2v^2\cos^2\theta) = -x^2$	**M1**		
	$v = 5$ m s^{-1}	**A1**	3	Accept 4.99
(iii)	$V^2 = (v\cos\theta)^2 + (v\sin\theta)^2 + 2 \times 8g$	**M1**		
	$V = 13.6$ m s^{-1}	**A1**	2	

Q43

7 (i)	$a = 14k - 0.8(1 + k^2)$ and $2a = 42k - 7.2(1 + k^2)$	**M1**		Creates 2 simultaneous equations
	$42k - 7.2(1 + k^2) = 2[14k - 0.8(1 + k^2)]$	**M1**		Creates a single equation in k
	$k = 1/2$ and 2	**B1**		Both values
	$\theta = \tan^{-1}k$	**M1**		With 1 of the candidates value of k
	$\theta = 63.435$ AG	**A1**	5	
(ii)	$t = 14/(35\cos 63.435)$	**M1**		
	$t\ (= 0.89442..) = 0.894$ s	**A1**	2	

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(iii)	$V_v = 35\sin63.4 - g[42/(35\cos63.4)]$ $\tan\alpha = 4.495/(35\cos63.4)$	**M1**		$V_v = 4.495$
	$\alpha = 15.9°$ above the horizontal	**A1**		Accept 16(.0)°
	$V^2 = 4.495^2 + (35\cos63.4)^2$	**M1**		
	$V = 16.3\ \text{m}s^{-1}$	**A1**	4	
	OR			
	$2a = 48$ $V^2 = 35^2 - 2g \times 48$	**M1**		$42 \times 2 - 7.2(1 + 2^2)$
	$V = 16.3\ \text{m}s^{-1}$	**A1**		
	$\cos\alpha = 35\cos63.435/16.3$	**M1**		
	$\alpha = 15.9°$	**A1**	4	

Q44

1	$Y = g1.6^2/2$	**B1**		12.8 m
	$12.8/(1.6V) = \tan30$	**M1**		$1.6V = X = 22.17$ m
	$V = 13.9\,\text{m}\,\text{s}^{-1}$	**A1**	[3]	

Q45

4 (i)	$-20\sin30 = 20\sin30 - gT$	**M1**		
	$T = 2$ s	**A1**		
	$OA = 34.6$ m	**B1**	[3]	
(ii)	$V_v^2 = 18^2 - (20\cos30)^2$	**M1**		
	$V_V = (\pm)\ 4.899$	**A1**		
	$4.899 = 20\sin30 - gt$	**M1**		
	$t = 0.51(0)$ s	**A1**		
	$-4.899 = 8 - gt$ $t = 1.29$ $T = 3.29$ s	**A1**	[5]	

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Equilibrium of a Rigid Body

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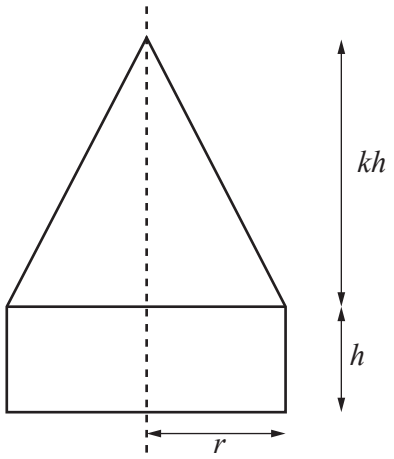
Q1

4



A uniform solid circular cone has vertical height kh and radius r. A uniform solid cylinder has height h and radius r. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram, which shows a cross-section). The cone and the cylinder are made of the same material.

(a) Show that the distance of the centre of mass of the combined solid from the base of the cylinder is $\frac{h(k^2+4k+6)}{4(3+k)}$. [4]

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The solid is placed on a plane that is inclined to the horizontal at an angle θ. The base of the cylinder is in contact with the plane. The plane is sufficiently rough to prevent sliding. It is given that $3h = 2r$ and that the solid is on the point of toppling when $\tan\theta = \frac{4}{3}$.

(b) Find the value of k. [3]

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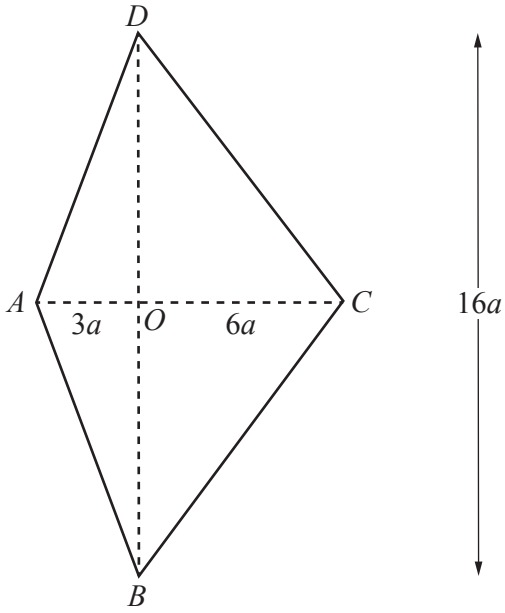
Q2

1



A uniform lamina $ABCD$ consists of two isosceles triangles ABD and BCD. The diagonals of $ABCD$ meet at the point O. The length of AO is $3a$, the length of OC is $6a$ and the length of BD is $16a$ (see diagram).

Find the distance of the centre of mass of the lamina from DB. [3]

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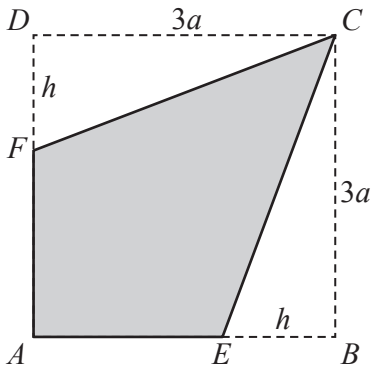
Q3

4



A uniform lamina $AECF$ is formed by removing two identical triangles BCE and CDF from a square lamina $ABCD$. The square has side $3a$ and $EB = DF = h$ (see diagram).

(a) Find the distance of the centre of mass of the lamina $AECF$ from AD and from AB, giving your answers in terms of a and h. [5]

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The lamina $AECF$ is placed vertically on its edge AE on a horizontal plane.

(b) Find, in terms of a, the set of values of h for which the lamina remains in equilibrium. [3]

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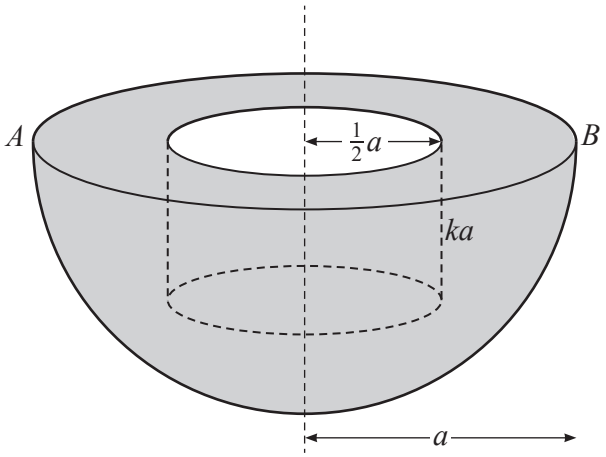
Q4

4



An object is formed by removing a solid cylinder, of height ka and radius $\frac{1}{2}a$, from a uniform solid hemisphere of radius a. The axes of symmetry of the hemisphere and the cylinder coincide and one circular face of the cylinder coincides with the plane face of the hemisphere. AB is a diameter of the circular face of the hemisphere (see diagram).

(a) Show that the distance of the centre of mass of the object from AB is $\dfrac{3a(2-k^2)}{2(8-3k)}$. [4]

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When the object is freely suspended from the point A, the line AB makes an angle θ with the downward vertical, where $\tan\theta = \frac{7}{18}$.

(b) Find the possible values of k. [3]

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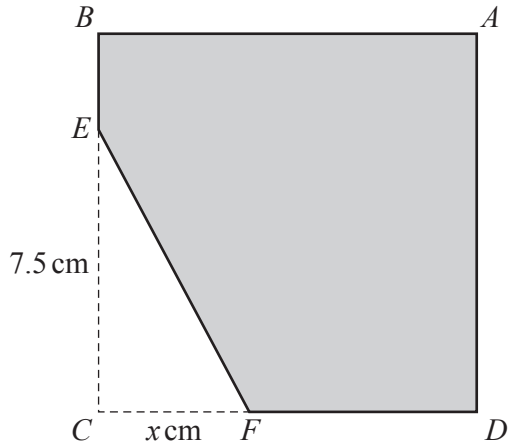
Q5

4



A uniform square lamina $ABCD$ has sides of length 10 cm. The point E is on BC with $EC = 7.5$ cm, and the point F is on DC with $CF = x$ cm. The triangle EFC is removed from $ABCD$ (see diagram). The centre of mass of the resulting shape $ABEFD$ is a distance $\bar{x}$ cm from CB and a distance $\bar{y}$ cm from CD.

(a) Show that $\bar{x} = \dfrac{400 - x^2}{80 - 3x}$ and find a corresponding expression for $\bar{y}$. [4]

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The shape *ABEFD* is in equilibrium in a vertical plane with the edge *DF* resting on a smooth horizontal surface.

(b) Find the greatest possible value of x, giving your answer in the form $a+b\sqrt{2}$, where a and b are constants to be determined. [3]

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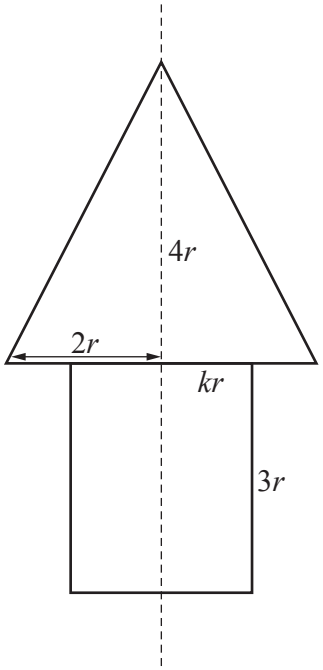
4



A uniform solid circular cone, of vertical height $4r$ and radius $2r$, is attached to a uniform solid cylinder, of height $3r$ and radius kr, where k is a constant less than 2. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram). The cone and the cylinder are made of the same material.

(a) Show that the distance of the centre of mass of the combined solid from the vertex of the cone is $\dfrac{(99k^2+96)r}{18k^2+32}$. [4]

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The point C is on the circumference of the base of the cone. When the combined solid is freely suspended from C and hanging in equilibrium, the diameter through C makes an angle α with the downward vertical, where $\tan\alpha = \frac{1}{8}$.

(b) Given that the centre of mass of the combined solid is within the cylinder, find the value of k. [4]

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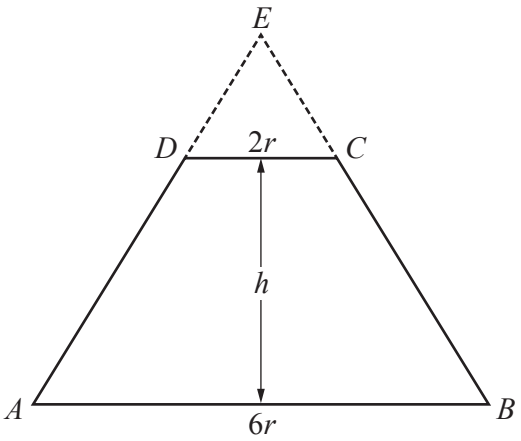
Q7

4



The diagram shows the cross-section $ABCD$ of a uniform solid object which is formed by removing a cone with cross-section DCE from the top of a larger cone with cross-section ABE. The perpendicular distance between AB and DC is h, the diameter AB is $6r$ and the diameter DC is $2r$.

(a) Find an expression, in terms of h, for the distance of the centre of mass of the solid object from AB. [4]

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The object is freely suspended from the point B and hangs in equilibrium. The angle between AB and the downward vertical through B is θ.

(b) Given that $h = \frac{13}{4}r$, find the value of $\tan\theta$. [2]

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Q8

3 An object consists of a uniform solid circular cone, of vertical height $4r$ and radius $3r$, and a uniform solid cylinder, of height $4r$ and radius $3r$. The circular base of the cone and one of the circular faces of the cylinder are joined together so that they coincide. The cone and the cylinder are made of the same material.

(a) Find the distance of the centre of mass of the object from the end of the cylinder that is not attached to the cone. [4]

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(b) Show that the object can rest in equilibrium with the curved surface of the cone in contact with a horizontal surface. [3]

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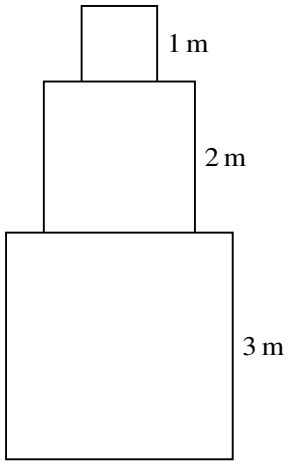
Q9

2



A uniform object is made by joining together three solid cubes with edges 3 m, 2 m and 1 m. The object has an axis of symmetry, with the cubes stacked vertically and the cube of edge 2 m between the other two cubes (see diagram).

(i) Calculate the distance of the centre of mass of the object above the base of the largest cube. [3]

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The smallest cube is now removed from the object. It is replaced by a heavier uniform cube with 1 m edges which is made of a different material. The centre of mass of the object is now at the base of the 2 m cube.

(ii) Find the ratio of the masses of the two cubes of edge 1 m. [3]

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Q10

6

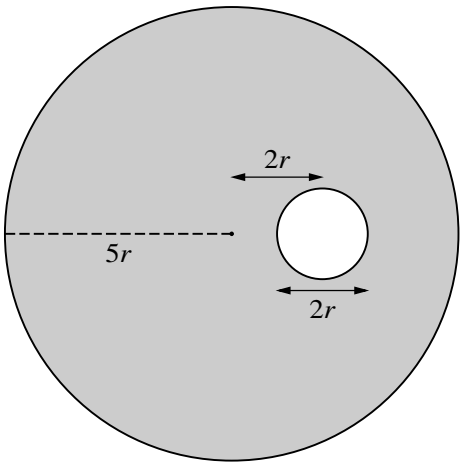

Fig. 1

Fig. 1 shows the cross-section of a solid cylinder through which a cylindrical hole has been drilled to make a uniform prism. The radius of the cylinder is $5r$ and the radius of the hole is r. The centre of the hole is a distance $2r$ from the centre of the cylinder.

(i) Find, in terms of r, the distance of the centre of mass of the prism from the centre of the cylinder. [4]

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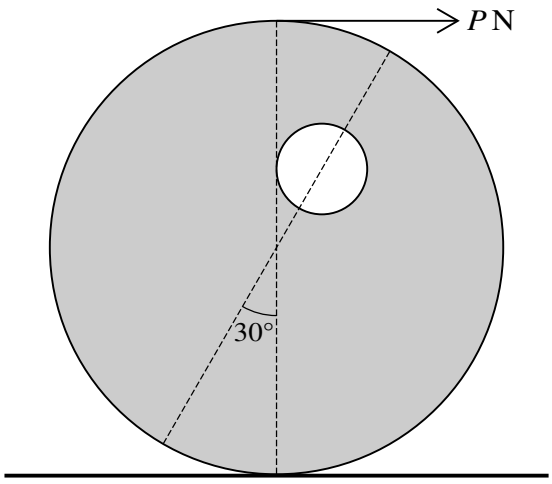

Fig. 2

The prism has weight W N and is placed with its curved surface on a rough horizontal plane. The axis of symmetry of the cross-section makes an angle of 30° with the vertical. A horizontal force of magnitude P N acting in the plane of the cross-section through the centre of mass is applied to the cylinder at the highest point of this cross-section (see Fig. 2). The prism rests in limiting equilibrium.

(ii) Find the coefficient of friction between the prism and the plane. [4]

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Q11

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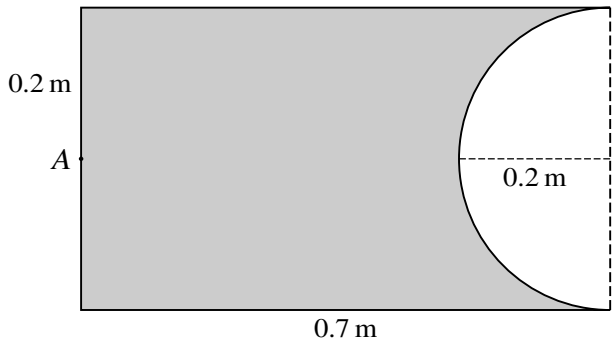

The diagram shows the cross-section through the centre of mass of a uniform solid object. The object is a cylinder of radius 0.2 m and length 0.7 m, from which a hemisphere of radius 0.2 m has been removed at one end. The point A is the centre of the plane face at the other end of the object. Find the distance of the centre of mass of the object from A. [5]

[The volume of a hemisphere is $\frac{2}{3}\pi r^3$.]

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Q12

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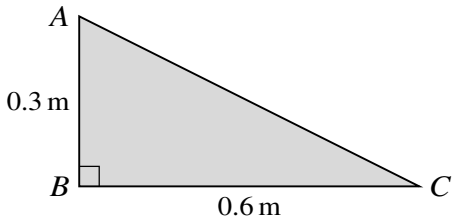

ABC is a uniform lamina in the form of a triangle with $AB = 0.3$ m, $BC = 0.6$ m and a right angle at B (see diagram).

(i) State the distances of the centre of mass of the lamina from AB and from BC. [2]

Distance from AB .....

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Distance from BC .....

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The lamina is freely suspended at B and hangs in equilibrium.

(ii) Find the angle between AB and the horizontal. [2]

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A force of magnitude 12 N is applied along the edge AC of the lamina in the direction from A towards C. The lamina, still suspended at B, is now in equilibrium with AB vertical.

(iii) Calculate the weight of the lamina. [3]

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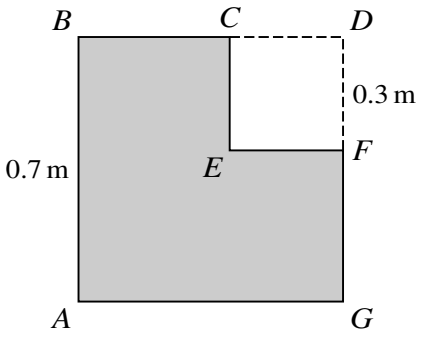
Q13

2



A uniform lamina $ABCEFG$ is formed from a square $ABDG$ by removing a smaller square $CDFE$ from one corner. $AB = 0.7$ m and $DF = 0.3$ m (see diagram). Find the distance of the centre of mass of the lamina from A. [4]

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Q14

7

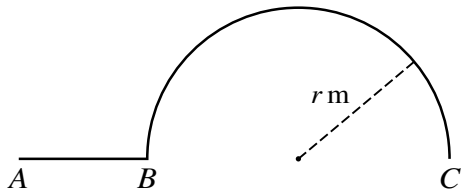

Fig. 1

Fig. 1 shows an object made from a uniform wire of length 0.8 m. The object consists of a straight part AB, and a semicircular part BC such that A, B and C lie in the same straight line. The radius of the semicircle is r m and the centre of mass of the object is 0.1 m from line ABC.

(i) Show that $r = 0.2$. [3]

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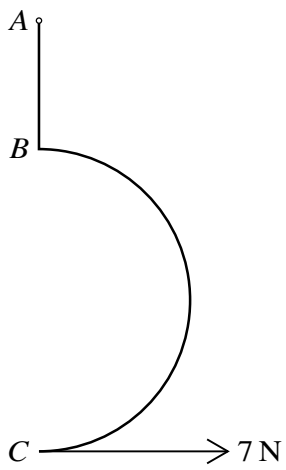

Fig. 2

The object is freely suspended at A and a horizontal force of magnitude 7 N is applied to the object at C so that the object is in equilibrium with ABC vertical (see Fig. 2).

(ii) Calculate the weight of the object. [3]

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[Question 7(iii) is printed on the next page.]

The 7 N force is removed and the object hangs in equilibrium with ABC at an angle of $\theta°$ with the vertical.

(iii) Find θ. [6]

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Q15

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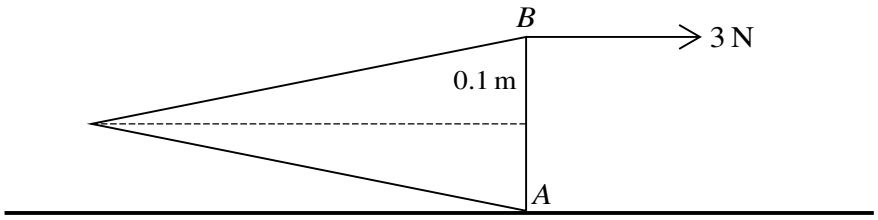

A uniform solid cone has weight 5 N and base radius 0.1 m. AB is a diameter of the base of the cone. The cone is held in equilibrium, with A in contact with a rough horizontal surface and AB vertical, by a force applied at B. This force has magnitude 3 N and acts parallel to the axis of the cone (see diagram). Calculate the height of the cone. [3]

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Q16

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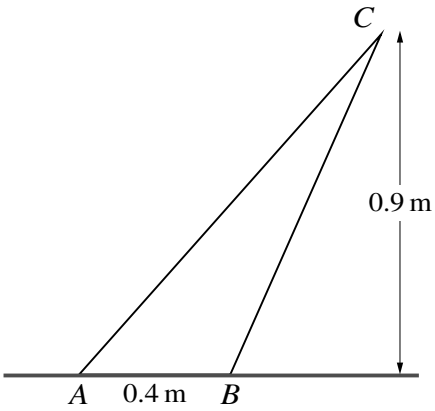

ABC is the cross-section through the centre of mass of a uniform prism which rests with *AB* on a rough horizontal surface. $AB = 0.4$ m and *C* is 0.9 m above the surface (see diagram). The prism is on the point of toppling about its edge through *B*.

(i) Show that angle $BAC = 48.4°$, correct to 3 significant figures. [3]

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A force of magnitude 18 N acting in the plane of the cross-section and perpendicular to AC is now applied to the prism at C. The prism is on the point of rotating about its edge through A.

(ii) Calculate the weight of the prism. [3]

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(iii) Given also that the prism is on the point of slipping, calculate the coefficient of friction between the prism and the surface. [4]

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Q17

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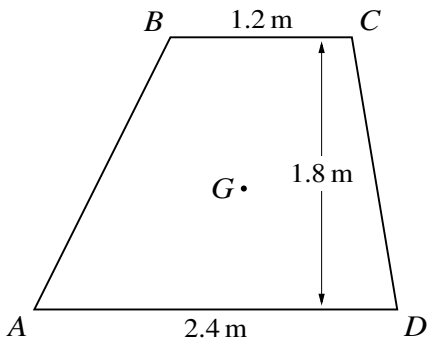

ABCD is a uniform lamina in the shape of a trapezium which has centre of mass *G*. The sides *AD* and *BC* are parallel and 1.8 m apart, with *AD* = 2.4 m and *BC* = 1.2 m (see diagram).

(i) Show that the distance of *G* from *AD* is 0.8 m. [4]

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The lamina is freely suspended at *A* and hangs in equilibrium with *AD* making an angle of 30° with the vertical.

(ii) Calculate the distance *AG*. [2]

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With the lamina still freely suspended at A a horizontal force of magnitude 7 N acting in the plane of the lamina is applied at D. The lamina is in equilibrium with AG making an angle of 10° with the downward vertical.

(iii) Find the two possible values for the weight of the lamina. [5]

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Q18

1 A uniform rectangular block has a square base $ABCD$ with $AB = BC = 0.4$ m. The height of the block is h m. The block is placed with its base on a rough plane inclined at 30° to the horizontal. The block does not slide. It is given that the block is on the point of toppling when the diagonal AC lies along a line of greatest slope. Calculate h. [3]

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Q19

7

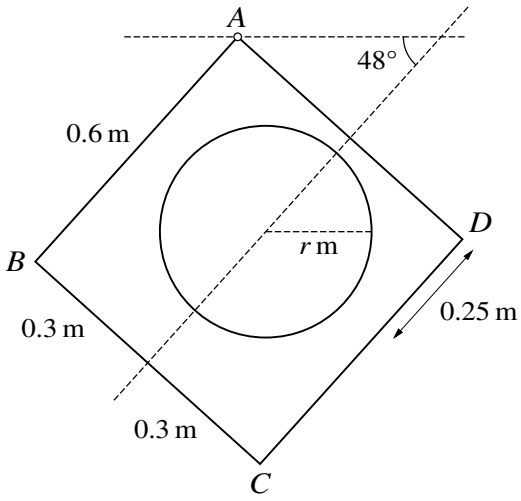

Fig. 1

ABCD is a uniform square lamina with sides of length 0.6 m. A circular hole of radius r m is made in the lamina. The centre of the hole is 0.3 m from *AB* and 0.25 m from *AD*. The lamina is freely suspended at *A* and hangs with the axis of symmetry making an angle of 48° with the horizontal (see Fig. 1).

(i) Show that $r = 0.214$, correct to 3 significant figures. [5]

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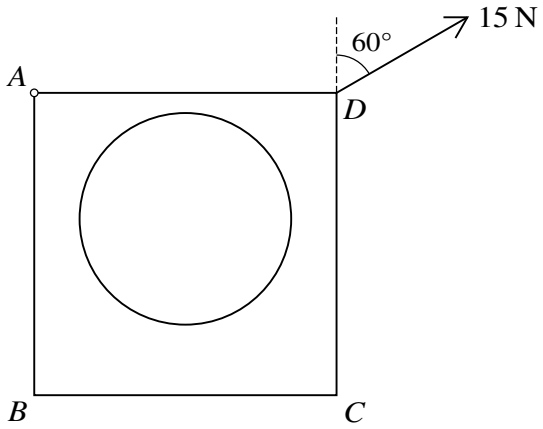

Fig. 2

The lamina is held in equilibrium with AD horizontal by a force of magnitude 15 N acting in the plane of the lamina applied at D. The line of action of this force makes an angle of 60° with the vertical (see Fig. 2).

(ii) Find the weight of the original square lamina, before the hole was made. [4]

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Q20

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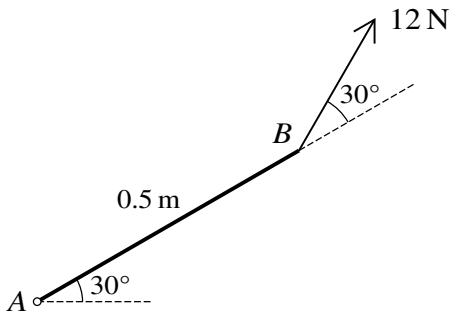

A non-uniform rod AB of length 0.5 m and weight 8 N is freely hinged to a fixed point at A. The rod makes an angle of 30° with the horizontal with B above the level of A. The rod is held in equilibrium by a force of magnitude 12 N acting in the vertical plane containing the rod at an angle of 30° to AB applied at B (see diagram). Find the distance of the centre of mass of the rod from A. [3]

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Q21

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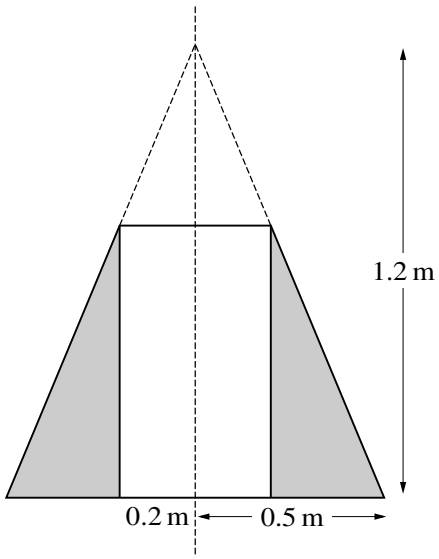

A uniform solid cone has height 1.2 m and base radius 0.5 m. A uniform object is made by drilling a cylindrical hole of radius 0.2 m through the cone along the axis of symmetry (see diagram).

(i) Show that the height of the object is 0.72 m and that the volume of the cone removed by the drilling is $0.0352\pi\,\text{m}^3$. [4]

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

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(ii) Find the distance of the centre of mass of the object from its base. [6]

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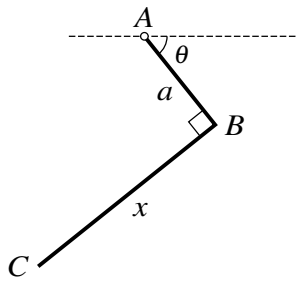
Q22

3



ABC is an object made from a uniform wire consisting of two straight portions AB and BC, in which $AB = a$, $BC = x$ and angle $ABC = 90°$. When the object is freely suspended from A and in equilibrium, the angle between AB and the horizontal is θ (see diagram).

(i) Show that $x^2 \tan\theta - 2ax - a^2 = 0$. [3]

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(ii) Given that $\tan\theta = 1.25$, calculate the length of the wire in terms of a. [2]

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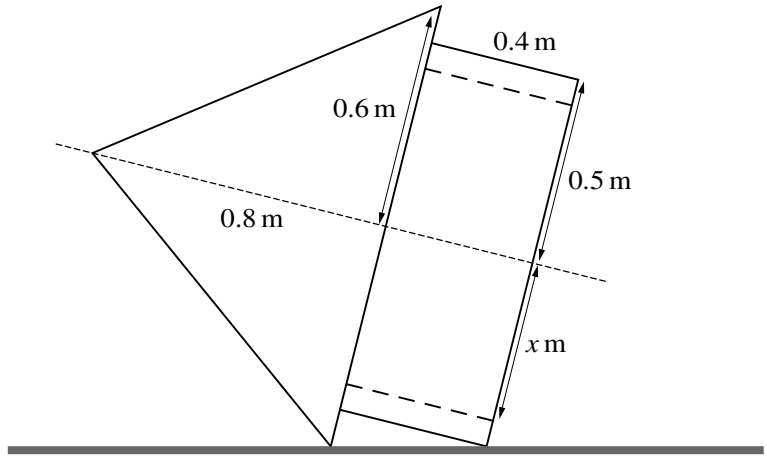
Q23

5



A uniform object is made by joining a solid cone of height 0.8 m and base radius 0.6 m and a cylinder. The cylinder has length 0.4 m and radius 0.5 m. The cylinder has a cylindrical hole of length 0.4 m and radius x m drilled through it along the axis of symmetry. A plane face of the cylinder is attached to the base of the cone so that the object has an axis of symmetry perpendicular to its base and passing through the vertex of the cone. The object is placed with points on the base of the cone and the base of the cylinder in contact with a horizontal surface (see diagram). The object is on the point of toppling.

(i) Show that the centre of mass of the object is 0.15 m from the base of the cone. [3]

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(ii) Find x. [4]

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

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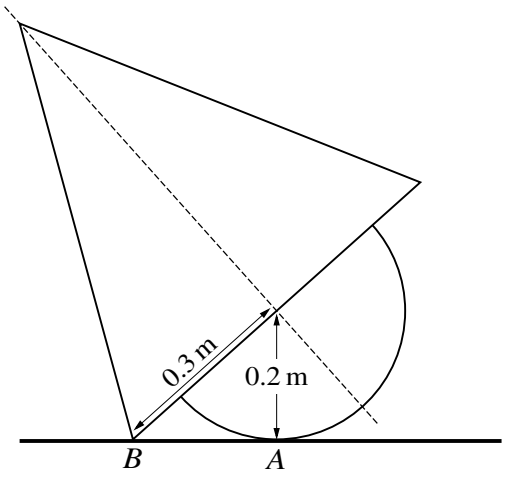
Q24

2



A uniform object is made by attaching the base of a solid hemisphere to the base of a solid cone so that the object has an axis of symmetry. The base of the cone has radius 0.3 m, and the hemisphere has radius 0.2 m. The object is placed on a horizontal plane with a point A on the curved surface of the hemisphere and a point B on the circumference of the cone in contact with the plane (see diagram).

(i) Given that the object is on the point of toppling about B, find the distance of the centre of mass of the object from the base of the cone. [3]

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(ii) Given instead that the object is on the point of toppling about A, calculate the height of the cone. [3]

[The volume of a cone is $\frac{1}{3}\pi r^2 h$. The volume of a hemisphere is $\frac{2}{3}\pi r^3$.]

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Q25

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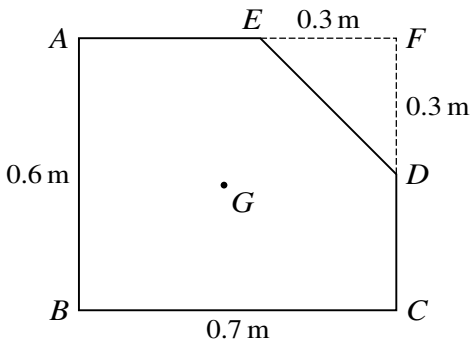

Fig. 1

Fig. 1 shows the cross-section $ABCDE$ through the centre of mass G of a uniform prism. The cross-section consists of a rectangle $ABCF$ from which a triangle DEF has been removed; $AB = 0.6$ m, $BC = 0.7$ m and $DF = EF = 0.3$ m.

(i) Show that the distance of G from BC is 0.276 m, and find the distance of G from AB. [5]

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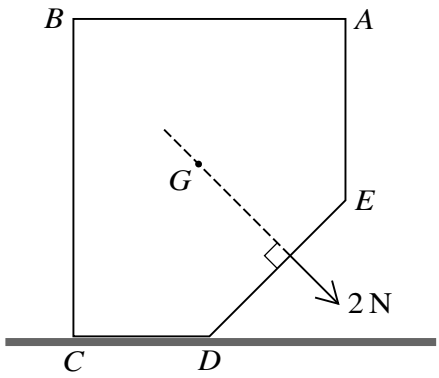

Fig. 2

The prism is placed with *CD* on a rough horizontal surface. A force of magnitude 2 N acting in the plane of the cross-section is applied to the prism. The line of action of the force passes through *G* and is perpendicular to *DE* (see Fig. 2). The prism is on the point of toppling about the edge through *D*.

(ii) Calculate the weight of the prism. [3]

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Q26

2 A uniform solid object is made by attaching a cone to a cylinder so that the circumferences of the base of the cone and a plane face of the cylinder coincide. The cone and the cylinder each have radius 0.3 m and height 0.4 m.

(i) Calculate the distance of the centre of mass of the object from the vertex of the cone. [4]

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

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The object has weight W N and is placed with its plane circular face on a rough horizontal surface. A force of magnitude kW N acting at 30° to the upward vertical is applied to the vertex of the cone. The object does not slip.

(ii) Find the greatest possible value of k for which the object does not topple. [3]

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Q27

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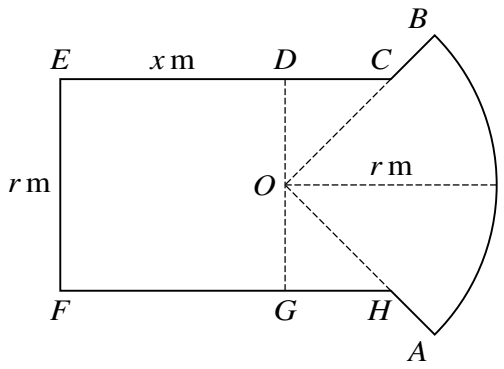

The diagram shows a uniform lamina $ABCDEFGH$. The lamina consists of a quarter-circle OAB of radius r m, a rectangle $DEFG$ and two isosceles right-angled triangles COD and GOH. The rectangle has $DG = EF = r$ m and $DE = FG = x$ m.

(i) Given that the centre of mass of the lamina is at O, express x in terms of r. [6]

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(ii) Given instead that the rectangle $DEFG$ is a square with edges of length r m, state with a reason whether the centre of mass of the lamina lies within the square or the quarter-circle. [1]

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Q28

2 A cylindrical container is open at the top. The curved surface and the circular base of the container are both made from the same thin uniform material. The container has radius 0.2 m and height 0.9 m.

(i) Show that the centre of mass of the container is 0.405 m from the base. [3]

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The container is placed with its base on a rough inclined plane. The container is in equilibrium on the point of slipping down the plane and also on the point of toppling.

(ii) Find the coefficient of friction between the container and the plane. [3]

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Q29

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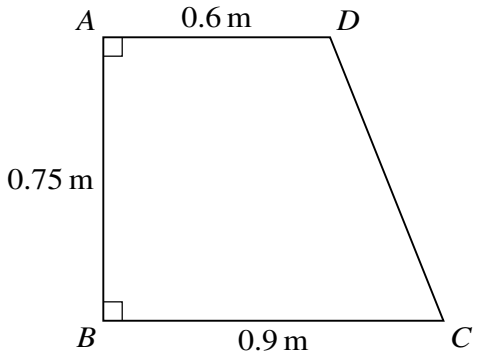

The diagram shows a uniform lamina $ABCD$ with $AB = 0.75$ m, $AD = 0.6$ m and $BC = 0.9$ m. Angle BAD = angle $ABC = 90°$.

(i) Show that the distance of the centre of mass of the lamina from AB is 0.38 m, and find the distance of the centre of mass from BC. [5]

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The lamina is freely suspended at B and hangs in equilibrium.

(ii) Find the angle between BC and the vertical. [2]

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Q30

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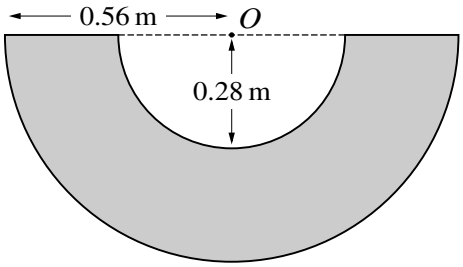

An object is made from a uniform solid hemisphere of radius 0.56 m and centre O by removing a hemisphere of radius 0.28 m and centre O. The diagram shows a cross-section through O of the object.

(i) Calculate the distance of the centre of mass of the object from O. [4]

[The volume of a hemisphere is $\frac{2}{3}\pi r^3$.]

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The object has weight 24 N. A uniform hemisphere H of radius 0.28 m is placed in the hollow part of the object to create a non-uniform hemisphere with centre O. The centre of mass of the non-uniform hemisphere is 0.15 m from O.

(ii) Calculate the weight of H. [3]

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Q31

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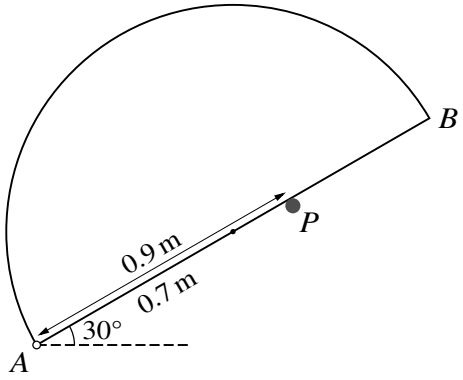

A uniform semicircular lamina of radius 0.7 m and weight 14 N has diameter AB. The lamina is in a vertical plane with A freely pivoted at a fixed point. The straight edge AB rests against a small smooth peg P above the level of A. The angle between AB and the horizontal is 30° and AP = 0.9 m (see diagram).

(i) Show that the magnitude of the force exerted by the peg on the lamina is 7.12 N, correct to 3 significant figures. [4]

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(ii) Find the angle with the horizontal of the force exerted by the pivot on the lamina at A. [3]

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Q32

3 An open box in the shape of a cube with edges of length 0.2 m is placed with its base horizontal and its four sides vertical. The four sides and base are uniform laminas, each with weight 3 N.

(i) Calculate the height of the centre of mass of the box above its base. [3]

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The box is now fitted with a thin uniform square lid of weight 3 N and with edges of length 0.2 m. The lid is attached to the box by a hinge of length 0.2 m and weight 2 N. The lid of the box is held partly open.

(ii) Find the angle which the lid makes with the horizontal when the centre of mass of the box (including the lid and hinge) is 0.12 m above the base of the box. [4]

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Q33

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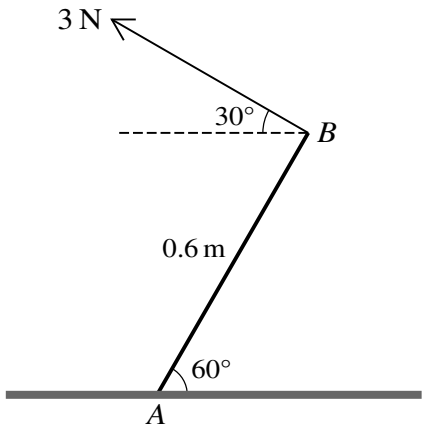

The end A of a non-uniform rod AB of length 0.6 m and weight 8 N rests on a rough horizontal plane, with AB inclined at 60° to the horizontal. Equilibrium is maintained by a force of magnitude 3 N applied to the rod at B. This force acts at 30° above the horizontal in the vertical plane containing the rod (see diagram).

(i) Find the distance of the centre of mass of the rod from A. [2]

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The 3 N force is removed, and the rod is held in equilibrium by a force of magnitude P N applied at B, acting in the vertical plane containing the rod, at an angle of 30° below the horizontal.

(ii) Calculate P. [2]

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In one of the two situations described, the rod AB is in limiting equilibrium.

(iii) Find the coefficient of friction at A. [4]

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Q34

6 A solid object consists of a uniform hemisphere of radius 0.4 m attached to a uniform cylinder of radius 0.4 m so that the circumferences of their circular faces coincide. The hemisphere and cylinder each have weight 20 N. The centre of mass of the object lies at the centre O of their common circular face.

(i) Show that the height of the cylinder is 0.3 m. [2]

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A new object is made by cutting the cylinder in half and removing the half not attached to the hemisphere. The cut is perpendicular to the axis of symmetry, so the new object consists of a hemisphere and a cylinder half the height of the original cylinder.

(ii) Find the distance of the centre of mass of the new object from O. [4]

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The new object is placed with its hemispherical part on a rough horizontal surface. The new object is held in equilibrium by a force of magnitude P N acting along its axis of symmetry, which is inclined at 30° to the horizontal.

(iii) Find P. [3]

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Q35

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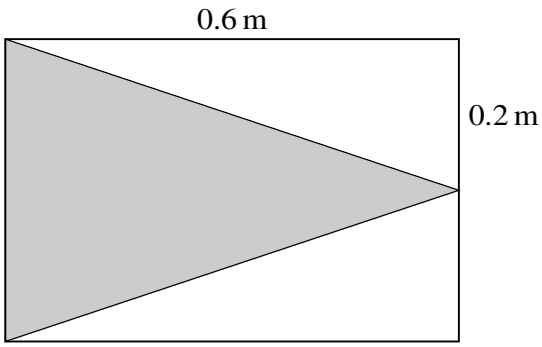

A uniform solid cone has height 0.6 m and base radius 0.2 m. A uniform hollow cylinder, open at both ends, has the same dimensions. An object is made by putting the cone inside the cylinder so that the base of the cone coincides with one end of the cylinder (see diagram, which shows a cross-section). The total weight of the object is 60 N and its centre of mass is 0.25 m from the base of the cone. Calculate the weight of the cone. [3]

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Q36

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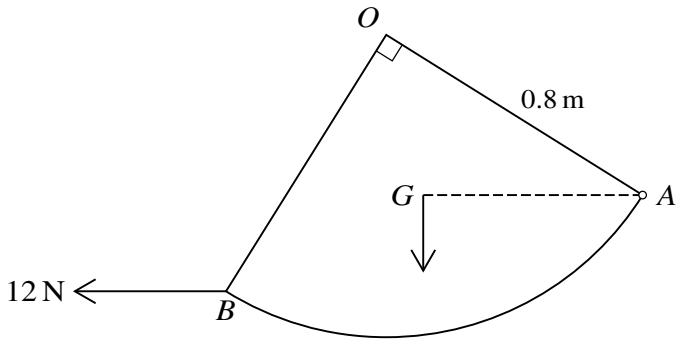

OAB is a uniform lamina in the shape of a quadrant of a circle with centre *O* and radius 0.8 m which has its centre of mass at *G*. The lamina is smoothly hinged at *A* to a fixed point and is free to rotate in a vertical plane. A horizontal force of magnitude 12 N acting in the plane of the lamina is applied to the lamina at *B*. The lamina is in equilibrium with *AG* horizontal (see diagram).

(i) Calculate the length *AG*. [3]

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(ii) Find the weight of the lamina. [5]

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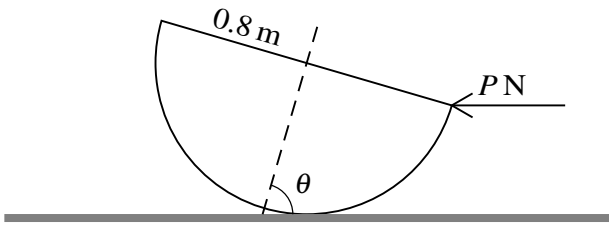

A uniform solid hemisphere of weight 60 N and radius 0.8 m rests in limiting equilibrium with its curved surface on a rough horizontal plane. The axis of symmetry of the hemisphere is inclined at an angle of θ to the horizontal, where $\cos\theta = 0.28$. Equilibrium is maintained by a horizontal force of magnitude P N applied to the lowest point of the circular rim of the hemisphere (see diagram).

(i) Show that $P = 8.75$. [3]

(ii) Find the coefficient of friction between the hemisphere and the plane. [2]

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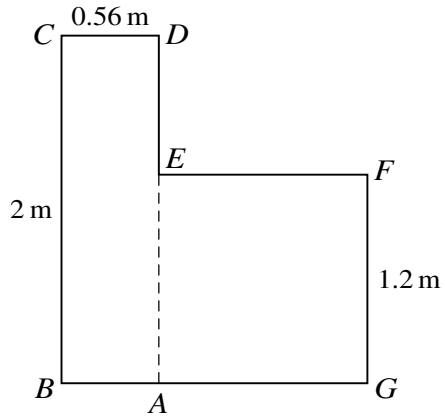

A uniform lamina is made by joining a rectangle $ABCD$, in which $AB = CD = 0.56$ m and $BC = AD = 2$ m, and a square $EFGA$ of side 1.2 m. The vertex E of the square lies on the edge AD of the rectangle (see diagram). The centre of mass of the lamina is a distance h m from BC and a distance v m from BAG.

(i) Find the value of h and show that $v = h$. [4]

The lamina is freely suspended at the point B and hangs in equilibrium.

(ii) State the angle which the edge BC makes with the horizontal. [1]

Instead, the lamina is now freely suspended at the point F and hangs in equilibrium.

(iii) Calculate the angle between FG and the vertical. [2]

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Q39

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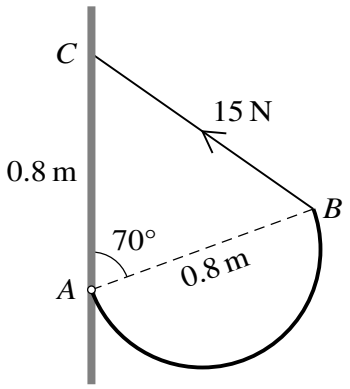

A uniform wire has the shape of a semicircular arc, with diameter AB of length 0.8 m. The wire is attached to a vertical wall by a smooth hinge at A. The wire is held in equilibrium with AB inclined at 70° to the upward vertical by a light string attached to B. The other end of the string is attached to the point C on the wall 0.8 m vertically above A. The tension in the string is 15 N (see diagram).

(i) Show that the horizontal distance of the centre of mass of the wire from the wall is 0.463 m, correct to 3 significant figures. [3]

(ii) Calculate the weight of the wire. [2]

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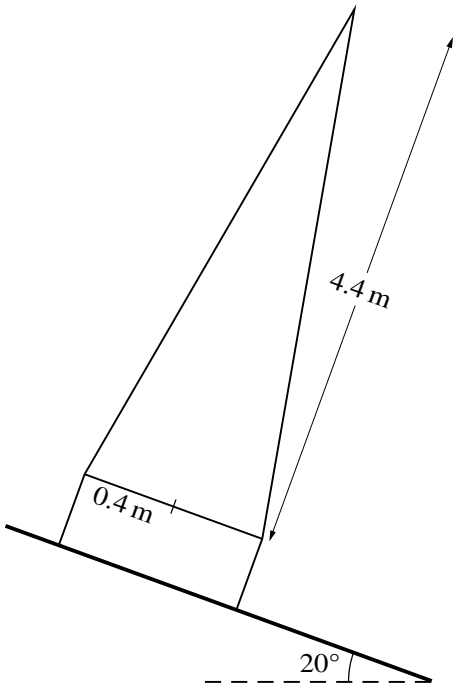
Q40

4



A uniform solid cone has base radius 0.4 m and height 4.4 m. A uniform solid cylinder has radius 0.4 m and weight equal to the weight of the cone. An object is formed by attaching the cylinder to the cone so that the base of the cone and a circular face of the cylinder are in contact and their circumferences coincide. The object rests in equilibrium with its circular base on a plane inclined at an angle of 20° to the horizontal (see diagram).

(i) Calculate the least possible value of the coefficient of friction between the plane and the object. [2]

(ii) Calculate the greatest possible height of the cylinder. [4]

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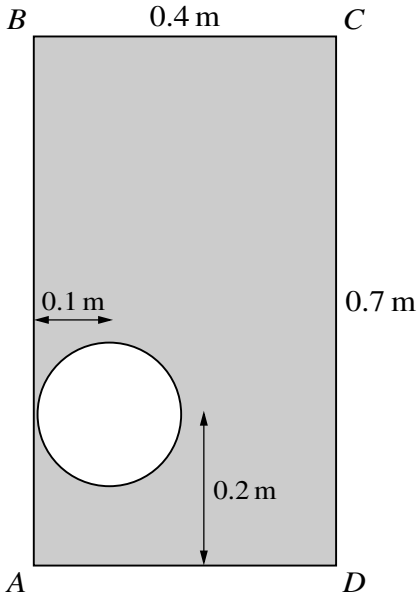

A uniform object is made by drilling a cylindrical hole through a rectangular block. The axis of the cylindrical hole is perpendicular to the cross-section $ABCD$ through the centre of mass of the object. $AB = CD = 0.7$ m, $BC = AD = 0.4$ m, and the centre of the hole is 0.1 m from AB and 0.2 m from AD (see diagram). The hole has a cross-section of area $0.03\ \text{m}^2$.

(i) Show that the distance of the centre of mass of the object from AB is 0.212 m, and calculate the distance of the centre of mass from AD. [4]

The object has weight 70 N and is placed on a rough horizontal surface, with AD in contact with the surface. A vertically upwards force of magnitude F N acts on the object at C. The object is on the point of toppling.

(ii) Find the value of F. [2]

The force acting at C is removed, and the object is placed on a rough plane inclined at an angle $\theta°$ to the horizontal. AD lies along a line of greatest slope, with A higher than D. The plane is sufficiently rough to prevent sliding, and the object does not topple.

(iii) Find the greatest possible value of θ. [2]

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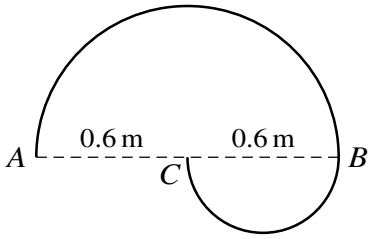

A uniform wire is bent to form an object which has a semicircular arc with diameter AB of length 1.2 m, with a smaller semicircular arc with diameter BC of length 0.6 m. The end C of the smaller arc is at the centre of the larger arc (see diagram). The two semicircular arcs of the wire are in the same plane.

(i) Show that the distance of the centre of mass of the object from the line ACB is 0.191 m, correct to 3 significant figures. [3]

The object is freely suspended at A and hangs in equilibrium.

(ii) Find the angle between ACB and the vertical. [4]

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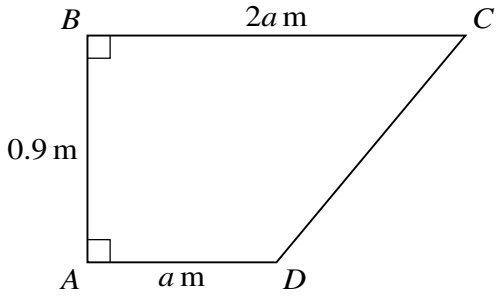
Q43

4



The diagram shows the cross-section $ABCD$ through the centre of mass of a uniform solid prism. $AB = 0.9$ m, $BC = 2a$ m, $AD = a$ m and angle ABC = angle $BAD = 90°$.

(i) Calculate the distance of the centre of mass of the prism from AD. [2]

(ii) Express the distance of the centre of mass of the prism from AB in terms of a. [2]

The prism has weight 18 N and rests in equilibrium on a rough horizontal surface, with AD in contact with the surface. A horizontal force of magnitude 6 N is applied to the prism. This force acts through the centre of mass in the direction BC.

(iii) Given that the prism is on the point of toppling, calculate a. [3]

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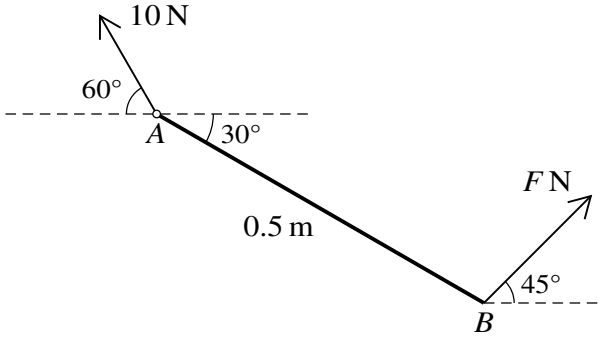

A non-uniform rod AB of length 0.5 m is freely hinged to a fixed point at A. The rod is in equilibrium at an angle of 30° with the horizontal with B below the level of A. Equilibrium is maintained by a force of magnitude F N applied at B acting at 45° above the horizontal in the vertical plane containing AB. The force exerted by the hinge on the rod has magnitude 10 N and acts at an angle of 60° above the horizontal (see diagram).

(i) By resolving horizontally and vertically, calculate F and the weight of the rod. [4]

(ii) Find the distance of the centre of mass of the rod from A. [3]

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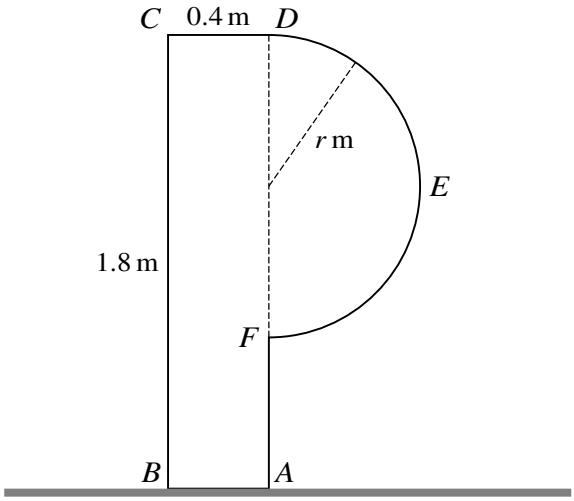
Q45

6



The diagram shows the cross-section $ABCDEF$ through the centre of mass of a uniform prism which rests with AB on rough horizontal ground. $ABCD$ is a rectangle with $AB = CD = 0.4$ m and $BC = AD = 1.8$ m. The other part of the cross-section is a semicircle with diameter DF and radius r m.

(i) Given that the prism is on the point of toppling, show that $r = 0.6$. [3]

A force of magnitude P N is applied to the prism, acting at 60° to the upwards vertical along a tangent to the semicircle at a point between D and E. The prism has weight 15 N and is in equilibrium on the point of toppling about B.

(ii) Show that $P = 3.26$, correct to 3 significant figures. [4]

(iii) Find the smallest possible value of the coefficient of friction between the prism and the ground. [2]

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Equilibrium of a Rigid Body MS

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Q1

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Volume</th><th>Centre of mass from base of cylinder</th></tr><tr><td>Cone</td><td>$\frac{1}{3}\pi r^2 kh$</td><td>$h+\frac{kh}{4}$</td></tr><tr><td>Cylinder</td><td>$\pi r^2 h$</td><td>$\frac{h}{2}$</td></tr><tr><td>Combined</td><td>$\pi r^2 h\left(\frac{1}{3}k+1\right)$</td><td>$\bar{x}$</td></tr></table>	**M1**	
	Take moments about base: $\pi r^2 h\left(\frac{1}{3}k+1\right)\bar{x}=\frac{1}{3}\pi r^2 kh\left(h+\frac{kh}{4}\right)+\pi r^2 h\ \frac{h}{2}$	**A1** **A1**	2 terms correct. All terms correct.
	$\bar{x}=\frac{h\left(k^2+4k+6\right)}{4(3+k)}$	**A1**	AG. Shown convincingly.
		4	
4(b)	$\tan\theta=\frac{r}{\bar{x}}$	**M1**	
	$\frac{4}{3}=\frac{6h(k+3)}{h\left(k^2+4k+6\right)}$ $2k^2-k-15=0$	**M1**	Equate to $\frac{4}{3}$ and simplify to quadratic.
	$k=3$	**A1**	CAO. No other solutions.
		3	

Q2

Question	Answer	Marks	Guidance
1	<table><tr><th></th><th>Area</th><th>Centre of mass from *DB*</th></tr><tr><td>*ABD*</td><td>$24a^2$</td><td>–</td></tr><tr><td>*BCD*</td><td>$48a^2$</td><td>$2a$</td></tr><tr><td>Combined</td><td>$72a^2$</td><td>$\bar{x}$</td></tr></table>	**B1**	All distances correct. *ABCD* can be split in other ways, for example *ADC* and *ABC*.
	Taking moments about *DB*: $72a^2\bar{x}=24a^2\times -a+48a^2\times 2a$ *OR* Taking moments about *A*: $72a^2\bar{x}=24a^2\times 2a+48a^2\times 5a$ *OR* Taking moments about *G:* $24a^2(\bar{x}+a)=48a^2\times(2a-\bar{x})$	**M1**	Moments equation with masses in correct ratio.
	$\bar{x}=a$	**A1**	CWO
	Alternative method for question 1		
	ADC: distance of centre of mass from BD = $\frac{6a-3a}{3}=a$ *ABC:* distance of centre of mass from BD = $\frac{6a-3a}{3}=a$	**B1**	One calculation.
	Second calculation or statement about symmetry	**M1**	
	$\bar{x}=a$	**A1**	
		3	

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Q3

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Area</th><th>Centre of mass from AD</th></tr><tr><td>Square</td><td>$9a^2$</td><td>$\frac{3}{2}a$</td></tr><tr><td>CDF</td><td>$\frac{3}{2}ah$</td><td>a</td></tr><tr><td>BEC</td><td>$\frac{3}{2}ah$</td><td>$3a-\frac{1}{3}h$</td></tr><tr><td>Resulting $AEFC$</td><td>$9a^2-3ah$</td><td>$\bar{x}$</td></tr></table>	**M1**	Attempt at moments with three terms.
	Taking moments about AD: $\left(9a^2-3ah\right)\ \bar{x}=\left(9a^2\times\frac{3}{2}a\right)-\left(\frac{3}{2}ah\times a\right)-\left(\frac{3}{2}ah\times\left(3a-\frac{1}{3}h\right)\right)$	**A1** **A1**	Two terms correct. All correct.
	$\bar{x}=\frac{27a^2-12ah+h^2}{6(3a-h)}\ \left(=\frac{9a-h}{6}\right)$	**A1**	AEF
	$\bar{y}=\bar{x}$	**B1**	By symmetry or equal to their $\bar{x}$.
		5	
4(b)	For equilibrium, $\bar{x} \leqslant 3a-h$ $27a^2-12ah+h^2 \leqslant 6(3a-h)^2$	**B1**	Accept strict inequality.
	$27a^2-24ah+5h^2 \geqslant 0$	**M1**	Homogeneous 3-term quadratic inequality.
	$h \leqslant \frac{9}{5}a$	**A1**	CAO.
		3	

Q4

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Volume</th><th>Centre of mass from AB</th></tr><tr><td>Hemisphere</td><td>$\frac{2}{3}\pi a^3$</td><td>$\frac{3}{8}a$</td></tr><tr><td>Cylinder</td><td>$\pi ka(\frac{a}{2})^2$</td><td>$\frac{ka}{2}$</td></tr><tr><td>Remainder</td><td>$\frac{2}{3}\pi a^3-\pi ka\left(\frac{a}{2}\right)^2$</td><td>$\bar{x}$</td></tr></table>	**M1**	Attempt at moments, 3 terms.
	Taking moments about AB: $\left(\frac{2}{3}\pi a^3-\pi ka\left(\frac{a}{2}\right)^2\right)\times\bar{x}=\left(\frac{2}{3}\pi a^3\times\frac{3}{8}a\right)-\left(\pi ka\left(\frac{a}{2}\right)^2\times\frac{ka}{2}\right)$	**A1** **A1**	Any 2 terms correct. All correct.
	$\bar{x}=\frac{3a\left(2-k^2\right)}{2(8-3k)}$	**A1**	Shown convincingly, AG.
		4	
4(b)	$\tan\theta=\frac{\bar{x}}{a}$ $\frac{3\left(2-k^2\right)}{2(8-3k)}=\frac{7}{18}$	**B1**	
	$27k^2-21k+2=0$	**M1**	Rearrange to form quadratic.
	$k=\frac{2}{3}$ and $k=\frac{1}{9}$	**A1**	Both answers correct.
		3	

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Q5

Question	Answer	Marks
4(a)	<table><tr><th></th><th>Area</th><th>Centre of mass from BC</th><th>Centre of mass from DC</th></tr><tr><td>Square</td><td>100</td><td>5</td><td>5</td></tr><tr><td>Triangle</td><td>x. 15/2</td><td>$\frac{1}{3}x$</td><td>$\frac{5}{2}$</td></tr><tr><td>Shape $ABEFD$</td><td>$100-\frac{15}{4}x$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table> Take moments about BC: $\left(100-\frac{15}{4}x\right)\sigma\,.\,\bar{x}=500\sigma-\frac{15}{4}x\sigma.\frac{1}{3}x$ (**M1** for all terms present)	**M1**
	$\bar{x}=\frac{400-x^2}{80-3x}$ **AG**	**A1**
	Take moments about DC: $\left(100-\frac{15}{4}x\right).\,\bar{y}\,=100\times5-\frac{15}{4}x.\frac{5}{2}$	**M1**
	$\bar{y}\,=\frac{800-15x}{160-6x}$	**A1**
		4
4(b)	Use condition: $\bar{x}\geqslant x$	**B1**
	$2x^2-80x+400\geqslant0$	**M1**
	$x=20-10\sqrt{2}$	**A1**
		3

Q6

Question	Answer	Marks
4(a)	<table><tr><th></th><th>Mass</th><th>Centre of mass from vertex of cone</th></tr><tr><td>Cylinder</td><td>$\pi(kr)^2.3r\rho$</td><td>$\frac{11}{2}r$</td></tr><tr><td>Cone</td><td>$\frac{1}{3}\pi(2r)^2.4r\rho$</td><td>$3r$</td></tr><tr><td>Combined</td><td>$\left(\pi(kr)^2.3r+\frac{1}{3}\pi(2r)^2.4r\right)\rho$</td><td>$\bar{x}$</td></tr></table>	**B1**
	Take moments about vertex: $\bar{x}\times\left(\pi(kr)^2.3r+\frac{1}{3}\pi(2r)^2.4r\right)=\frac{11r}{2}\times\ \pi(kr)^2.3r+3r\times\ \frac{1}{3}\pi(2r)^2.4r$ leading to	**M1A1**
	$\bar{x}=\frac{\left(99k^2+96\right)r}{18k^2+32}$	**A1**
		4
4(b)	$\tan\alpha=\frac{\bar{x}-4r}{2r}$ (= 1/8)	**M1**
	$\bar{x}=\frac{17}{4}r$	**A1**
	Equate to answer in **(a)** and attempt to find k	**M1**
	$45k^2=80:\ \ k=\frac{4}{3}$	**A1**
		4

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Q7

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Volume</th><th>Centre of mass from AB</th></tr><tr><td>Small cone</td><td>$\frac{1}{3}\pi r^2.\frac{h}{2}$</td><td>$h+\frac{1}{4}.\frac{h}{2}\left(=\frac{9h}{8}\right)$</td></tr><tr><td>Large cone</td><td>$\frac{1}{3}\pi(3r)^2.\frac{3h}{2}$</td><td>$\frac{1}{4}.\frac{3h}{2}\left(=\frac{3h}{8}\right)$</td></tr><tr><td>Object</td><td>$\frac{26}{6}\pi(r)^2 h$</td><td>$\bar{x}$</td></tr></table>	**B1**	For 9h/8 or 3h/8 (unsimplified)
	Take moments about AB $\frac{13}{3}\pi r^2 h.\bar{x}=\frac{27}{6}\pi r^2 h.\frac{3h}{8}-\frac{1}{6}\pi r^2 h.\frac{9h}{8}$	**M1 A1**	Moments equation: Allow use of relative masses 1, 26, 27
	$\bar{x}=\frac{9h}{26}$	**A1**	
		4	
4(b)	$\tan\theta=\frac{\bar{x}}{3r}$	**M1**	
	$(=\frac{3h}{26r})$ Use $h=\frac{13}{4}r$ $\tan\theta=\frac{3}{8}$	**A1**	
		2	

Q8

Question	Answer	Marks	Guidance
3(a)	<table><tr><th></th><th>Volume</th><th>Centre of mass from base</th></tr><tr><td>Cone</td><td>$\frac{1}{3}\pi(3r)^2.\ 4r$</td><td>$4r+r$</td></tr><tr><td>Cylinder</td><td>$\pi(3r)^2.\ 4r$</td><td>$2r$</td></tr><tr><td>Combined</td><td>$\frac{4}{3}\pi(3r)^2.\ 4r$</td><td>$\bar{x}$</td></tr></table>	**B1**	Distances correct
	Taking moments about base of cylinder: $\bar{x}.\frac{4}{3}\pi(3r)^2.4r=\frac{1}{3}\pi(3r)^2.4r.5r+\pi(3r)^2.4r.2r$	**M1 A1**	Moments equation
	$\bar{x}=\frac{11}{4}r$	**A1**	
		4	
3(b)	Condition: $OG\cos\theta<OA$ (where O is vertex of cone and OA is slant height of cone)	**B1**	Correct condition for equilibrium
	$\left(4r+\frac{5r}{4}\right)\times\frac{4}{5}<5r$	**M1**	Expression in terms of r
	$21 < 25$ True	**A1**	Correct conclusion, with correct working
		3	

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Q9

Question	Answer	Marks	Guidance
2(i)	Total volume (= 27 + 8 + 1) = 36	**B1**	
	$36x = 27\times1.5 + 8\times4 + 1\times5.5$	**M1**	Take moments about base of largest cube
	x (= 13/6) = 2.17 m	**A1**	
		3	
2(ii)	Mass of new cube = 35 + m	**B1**	Where m is the mass of the new cube
	(35 + m) × 3 =27×1.5 + 8×4 + 5.5m (leads to m = 13)	**M1**	Take moments about base of largest cube
	13:1 or 1:13	**A1**	Accept 13
		3	

Q10

Question	Answer	Marks	Guidance
6(i)	Area of hole = πr^2 and Area of original circle = $25\pi r^2$	**M1**	
	Area of cross-section = $24\pi r^2$	**A1**	
	πr^2 (2r) = $24\pi r^2$ (d)	**M1**	Take moments about the centre of the cylinder
	d = r/12 (= 0.083333....r)	**A1**	
		4	
6(ii)	P(2 × 5r) = W(r/12)cos60	**M1**	Take moments about the point of contact with the plane
	P = Wcos60/120 = W/240 = 0.00417W (= F)	**A1**	
	μ = (Wcos60/120)/W	**M1**	Use F = μR Note R = W by resolving vertically
	μ = 1/240 = 0.00417	**A1**	
		4	

Q11

Question	Answer	Marks	Guidance
3	Volume of cylinder = $\pi \times 0.22 \times 0.7$ (= 0.028π) **AND** Volume of hemisphere = $2\pi\times\frac{0.2^3}{3}\left(=0.0053333\pi\right)$	**B1**	Both volumes required for **B1**
	Distance of centre of mass from object base $=0.7-3\times\frac{0.2}{8}\left(=0.625\right)$	**B1**	
	$x\left(\pi\times0.2^2\times0.7-2\pi\times\frac{0.2^3}{3}\right)+\left(0.7-3\times\frac{0.2}{8}\right)\times2\pi\times\frac{0.2^3}{3}=0.35\times0.028\pi$	**M1A1**	Take moments about the plane face
	$x = 0.285$ m	**A1**	
		5	

Q12

Question	Answer	Marks	Guidance
6(i)	From $AB = 0.2$	**B1**	
	From $BC = 0.1$	**B1**	
		2	
6(ii)	$\tan\theta=\frac{0.1}{0.2}$	**M1**	θ is the angle between AB and the horizontal
	θ = 26.6°	**A1**	
		2	
6(iii)	$12\cos26.6\times0.3 = W\times0.2$	**M1A1**	Take moments about B. (W is the weight of the lamina)
	$W = 16.1\ N$	**A1**	
		3	

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Q13

Question	Answer	Marks	Guidance
2	Original square: Area = 0.7^2, CoM = $\sqrt{\left(0.35^2+0.35^2\right)}$ and Smaller square: Area = 0.3^2, CoM = $\sqrt{\left(0.15^2+0.15^2\right)}$	**B1**	0.49, 0.495 from A 0.09, 0.21213... from D or E
	$AX\left(0.49-0.09\right)+0.09\left(\sqrt{0.98}-\sqrt{0.045}\right)=0.49\times0.495$	**M1A1**	Attempt to take moments about A
	$AX=0.431$ m	**A1**	
		4	
	Alternative method for question 2		
	$(0.49\times0.35)=(0.09\times0.55)+0.4X\rightarrow X=0.305$	**M1**	Take moments about AG or AB
	$X=Y=0.305$	**B1**	
	$AX=\sqrt{\left(0.305^2+0.305^2\right)}$	**M1**	Use Pythagoras's theorem
	$AX=0.431$	**A1**	
		4	

Q14

Question	Answer	Marks	Guidance
7(i)	$X=\frac{2r}{\pi}$	**B1**	X = distance of centre of mass of the arc from ABC
	$0.8\times0.1=\pi r\times\frac{2r}{\pi}$	**M1**	Take moments about ABC
	$r=0.2$	**A1**	
		3	
7(ii)	$AC=0.8+2\times0.2-0.2\pi$ $(=0.57168\ldots)$	**B1**	
	$0.1W=7AC$	**M1**	AC must be a numerical value. Take moments about A
	$W=40(0.)N$	**A1**	
		3	
7(iii)	$(0.8-0.2\pi+0.2)$ [= 0.37168…]	**B1**	
	$0.8Y=\left(0.8-0.2\pi\right)\times\frac{\left(0.8-0.2\pi\right)}{2}+\left(0.2\pi\right)\times\left(0.8-0.2\pi+0.2\right)$	**M1A1**	
	$Y=0.310(338)$	**A1**	
	$\tan\theta=\frac{0.1}{0.310338}$	**M1**	
	$\theta=17.9$	**A1**	Allow 17.8

Q15

Question	Answer	Marks	Guidance
1	Conservation of momentum at $\frac{h}{4}$	**B1**	
	$\frac{5\times h}{4}=3\times0.2$	**M1**	Take moments about A
	$(h=)$ 0.48 m	**A1**	
		3	

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Q16

Question	Answer	Marks	Guidance
7(i)	$BG = 0.3$ m	**B1**	G is the CoM vertically above B. M is the mid-point of AB and E is v the point vertically below C on AB extended.
	$ME = 3 \times 0.2 = 0.6$ and $\tan A = \frac{CE}{AE} = \frac{0.9}{0.8}$	**M1**	Use of similar triangles and trigonometry of a right angled triangle
	$A = 48.4°$	**A1**	AG
		3	
7(ii)	$AC = \frac{0.9}{\sin 48.4} = 1.20(41...)$	**B1**	Use trigonometry of a right angled triangle
	$18 \times 1.2041 = 0.4W$	**M1**	Moments about A
	$W = 54.2$ N	**A1**	
		3	
7(iii)	$H = 18\sin A = 18\sin 48.4$ (= 13.46)	**B1**	Resolve horizontally
	$V = 54.2 - 18\cos 48.4$ (= 42.25)	**B1ft**	Resolve vertically
	$\mu = \frac{13.46}{42.25}$	**M1**	Use $F = \mu R$
	$\mu = 0.319$	**A1**	Accept 0.32
		4	

Q17

Question	Answer	Marks	Guidance
7(i)	Rectangle: Area $= 1.2 \times 1.8 = 2.16$, $y = \frac{1.8}{2} = 0.9$	**B1**	
	Triangle(s): Area $= 1.2 \times \frac{1.8}{2} = 1.08$, $y = \frac{1.8}{3} = 0.6$	**B1**	
	$(2.16 + 1.08)Y = 2.16 \times 0.9 + 1.08 \times 0.6$	**M1**	Take moments about AD
	$Y = 0.8$ m	**A1**	AG
		4	
7(ii)	$AG\sin 30 = 0.8$	**M1**	Use Trigonometry of a right angled triangle
	$AG = 1.6$ m	**A1**	
		2	
7(iii)	AD makes an angle of 40° or 20° with the vertical	**B1**	
	$W \times AG\sin 10 = 7 \times 2.4\cos 40$	**M1**	Take moments about A
	$W = 46.3$ N	**A1**	
	$W \times AG\sin 10 = 7 \times 2.4\cos 20$	**M1**	Take moments about A
	$W = 56.8$ N	**A1**	
		5	

Q18

Question	Answer	Marks	Guidance
1	$d = \sqrt{(0.2^2 + 0.2^2)}$ (= 0.2828) OR $AC = \sqrt{(0.4^2 + 0.4^2)}$ (= 0.56568..)	**B1**	Note $d = \frac{1}{2}AC$
	$\tan 30 = 0.2828 / (h / 2)$	**M1**	
	$h = 0.98(0)$	**A1**	$2\sqrt{6} / 5$
		3	

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Q19

Question	Answer	Marks	Guidance
7(i)	tan48 = $\bar{x}$ /0.3	**M1**	$\bar{x}$ is the distance of the centre of mass from *AD*
	$\bar{x}$ = 0.3332	**A1**	
	$0.6^2 \times 0.3 = \pi r^2 \times 0.25 + (0.6^2 - \pi r^2)\bar{x}$ OR $0.6^2 \times 0.3 = \pi r^2 \times 0.35 + (0.6^2 - \pi r^2) \times (0.6 - \bar{x})$	**M1**	Take moments about *AD* Take moments about *BC*
	$\pi r^2 \times (0.3332 - 0.25) = 0.6^2 \times (0.3332 - 0.3)$ OR $\pi r^2(0.6 - 0.3332 - 0.35) = 0.6^2 (0.6 - 0.3332 - 0.3)$	**A1**	
	r = 0.214 **AG**	**A1**	
		5	
7(ii)	$W = 0.6 \times 15\cos 60$	**M1**	Take moments about *C*.(W = weight of the lamina)
	W = 15	**A1**	
	Square = $15 \times 0.6^2 / (0.6^2 - \pi \times 0.214^2)$	**M1**	Recognise that the ratio of weights = ratio of areas
	Square = 25(.0) N	**A1**	
		4	

Q20

Question	Answer	Marks	Guidance
2		**M1**	Attempt to take moments about A
	$8x\cos 30 = 0.5 \times 12\sin 30$	**A1**	Correct equation
	x = 0.433 m	**A1**	
		3	

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Q21

Question	Answer	Marks	Guidance
7(i)	Height of conical tip $= 1.2 \times \frac{0.2}{0.5} = 0.48$	**M1**	Use ratio of corresponding sides, similar figures
	Cylindrical height = 1.2 – 0.48 = 0.72	**A1**	**AG**
	Volume removed $= \pi 0.2^2 \times \frac{0.48}{3} + \pi 0.2^2 \times 0.72$ $(= 0.0064\pi + 0.0288\pi)$	**M1**	
	Volume removed = 0.0352π	**A1**	**AG**
		4	

7(ii)	Moment of cone removed about the base $= 0.0064\pi(0.72 + \frac{0.48}{4}) = 0.0064\pi \times 0.84$	**B1**	
	Moment of cylinder removed about the base $= 0.0288\pi \times \frac{0.72}{2} = 0.0288\pi \times 0.36$	**B1**	
	Moment of the original cone about the base $= \pi\, 0.5^2 \times \frac{1.2}{3} \times 0.3 = 0.1\pi \times 0.3$	**B1**	
		M1	Attempt to take moments about the base
	$0.1\pi \times 0.3 = 0.0064\pi \times 0.84 + 0.0288\pi \times 0.36 +$ $0.0648\pi x$	**A1**	Note 0.0648π is the volume of the object
	$x = 0.22$ m	**A1**	
		6	

Q22

3(i)	d = $x\sin\theta/2 - a\cos\theta$ or equivalent	**B1**	Note d is the distance of the C of M of BC from the vertical through A
	$a(a\cos\theta)/2 = x(x\sin\theta/2 - a\cos\theta)$	**M1**	Take moments about A
	$x^2\tan\theta - 2ax - a^2 = 0$ **AG**	**A1**	
	Total:	**3**	
3(ii)	$1.25x^2 - 2ax - a^2 = 0$ [$x = 2a$ and $x = -2a/5$]	**M1**	Attempts to solve the equation
	Length (= 2a + a) = 3a	**A1**	
	Total:	**2**	

Q23

Question	Answer	Marks	Guidance
5(i)	$\tan\theta = (0.6 - 0.5)/0.4$ (= 1/4)	**B1**	θ is the angle made by the base and the vertical
	$\tan\theta = \bar{x}/0.6$	**M1**	
	$\bar{x} = 0.15$ m **AG**	**A1**	
	Total:	**3**	
5(ii)	$(\pi 0.6^2 \times 0.8/3) \times (0.8/4) - [\pi(0.5^2 - x^2) \times 0.4] \times (0.4/2)$ $= [\pi 0.6^2 \times 0.8/3 + \pi(0.5^2 - x^2) \times 0.4]\bar{x}$	**M1** **A1**	Attempts to take moments about the base of the cone using their $\bar{x}$ Note $\bar{x}=0.15$ Correct equation for the A mark.
		M1	Attempts to solve the equation
	$x = 0.464$	**A1**	Note $x^2 = 0.216$
	Total:	**4**	

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Q24

Question	Answer	Marks	Guidance
2(i)	$\theta = 0.2/0.3$	**B1**	Axis makes an angle θ with the horizontal
	$\tan\theta = x/0.3$	**M1**	
	$x = 0.335(41..)$	**A1**	
		3	
2(ii)		**M1**	Attempt to take moments about A
	$(\pi 0.3^2 h/3)\times(h/4) = (2\pi 0.2^3/3)(3\times 0.2/8)$	**A1**	
	$h = 0.231$	**A1**	
		3	

Q25

Question	Answer	Marks	Guidance
6(i)	Area of cross-section of prism $= 0.5\times 0.6 - 0.3\times 0.3/2 = 0.375\ m^2$	**B1**	Area of cross-section of prism = area of rectangle – area of triangle
	$0.375y = 0.42\times 0.6/2 - 0.045(0.6 - 0.3/3)$	**M1**	Take moments about BC
	$y = 0.276$ m **AG**	**A1**	
	$0.375x = 0.42\times 0.7/2 - 0.045(0.7 - 0.3/3)$	**M1**	Take moments about AB
	$x = 0.32$ m	**A1**	
		5	
6(ii)		**M1**	Attempt to take moments about D
	$2\cos 45\times (0.7 - 0.32) = 2\cos 45\times (0.3 - 0.276) + W(0.3 - 0.276)$	**A1**	
	$W = 21(.0)$ N	**A1**	
		3	

Q26

Question	Answer	Marks	Guidance
2(i)		**M1**	Attempt to take moments about the vertex of the cone
	$(\pi \times 0.3^2 \times 0.4/3) \times (3 \times 0.4/4) + (\pi \times 0.3^2 \times 0.4 \times (0.4 + 0.2))$	**A1**	
	$= (\pi \times 0.3^2 \times 0.4/3 + \pi \times 0.3^2 \times 0.4)\bar{x}$	**A1**	
	$\bar{x} = 0.525$ m	**A1**	
		4	
2(ii)		**M1**	Attempt to take moments about a point on the circumference of the base of the cone
	$kW\cos 30 \times 0.3 + kW\sin 30 \times 0.8 = 0.3W$	**A1**	
	$k = 0.455$	**A1**	
		3	

Q27

Question	Answer	Marks	Guidance
6(i)	Centre of mass of triangles below $O = r/6$	**B1**	
	Centre of mass of quadrant below $O = (2r\sin\pi/4)/(3\pi/4)$	**B1**	
		M1	Attempt to take moments about O
	$(rx)(x/2) = (r^2/4)(r/6) + (\pi r^2/4)(2r\sin\pi/4)/(3\pi/4)$	**A1**	
	$x^2 = 2\left(r^2/24 + r^2\sqrt{2}/3\right)$	**M1**	Attempt to express x in terms of r
	$x = 1.01r$	**A1**	
		6	

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6(ii)	Within quadrant as the square will be smaller than the rectangle **Or** if x < r in part **(i)**, within the square as the square will be larger than the rectangle	**B1ft**	
		1	

Q28

Question	Answer	Marks	Guidance
2(i)	$M = 2\pi$ x 0.2 x 0.9 + π x 0.2^2	**B1**	M = total mass of the container
	$(2\pi$ x 0.2 x 0.9 + π x $0.2^2)\,\bar{x}$ = 2π x 0.2 x 0.9 x 0.9/2	**M1**	Takes moments about the base
	$\bar{x}$ = 0.405 m AG	**A1**	
	Total:	**3**	
2(ii)	$\tan\theta = 0.2/0.405$	**M1**	θ is the angle of slope of the plane
	$\mu = \tan\theta$	**B1**	
	$\mu = 0.494$	**A1**	
	Total:	**3**	

Q29

Question	Answer	Marks	Guidance
4(i)	A = 0.6 x 0.75 + 0.3 x 0.75/2 (= 0.5625)	**B1**	A = total area of the lamina
	$0.5625\,\bar{x}$ = 0.75 x 0.6 x 0.3 + $\frac{1}{2}$0.3 x 0.75 x (0.6 + 0.3/3)	**M1**	Takes moments about AB
	$\bar{x}$ = 0.38 m (from AB) AG	**A1**	
	$0.5625\,\bar{y}$ = 0.75 x 0.6 x 0.375 + $\frac{1}{2}$0.3 x 0.75 x 0.25	**M1**	Takes moments about BC
	$\bar{y}$ = 0.35 m (from BC)	**A1**	
	Total:	**5**	
4(ii)	$\tan\theta = 0.35/0.38$	**M1**	$\tan\theta = \bar{y}/\bar{x}$ where θ is the required angle
	$\theta = 42.6$	**A1**	
	Total:	**2**	

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Q30

Question	Answer	Marks	Guidance
3(i)	CofM of hemisphere $= \frac{3}{8} \times 0.56$ or $\frac{3}{8} \times 0.28$	**B1**	
	$[\frac{2}{3}\pi \times 0.56^3 - \frac{2}{3}\pi \times 0.28^3]X = \frac{2}{3}\pi \times 0.56^3 \times \frac{3}{8} \times 0.56 - \frac{2}{3}\pi \times 0.28^3 \times \frac{3}{8} \times 0.28$	**M1A1**	Take moments about O
	$X = 0.225$ m	**A1**	
	Total:	**4**	
3(ii)	$24 \times 0.225 + W(3 \times 0.28 / 8) = (24 + W) \times 0.15$	**M1A1**	Attempts to take moments about O W = weight of uniform hemi-sphere
	$W = 40$ N	**A1**	
	Total:	**3**	

Q31

5(i)	OG $= 2 \times 0.7\sin(\pi / 2) / (3\pi / 2)$ (= 0.297)	**B1**	
	$0.9R = 14(0.7\cos30 - 0.297\sin30)$	**M1A1**	Attempts to take moments about A
	$R = 7.12$ N	**A1**	
	Total:	**4**	
5(ii)	H = 7.12sin30 and V = 14 − Rcos30	**M1**	Resolves horizontally and vertically
	$\tan\theta = (14 - 7.12\cos30) / (7.12\sin30)$	**M1**	Uses $\tan\theta$ = V / H, where θ is the required angle
	$\theta = 65.6$	**A1**	
	Total:	**3**	

Q32

3(i)	Height of C of M of each vertical face above the base = 0.1 m	**B1**	
	$5 \times 3y = 4 \times 3 \times 0.1$	**M1**	Takes moments about the base. y is the height of the C of M above the base
	$y = 0.08$ m	**A1**	
	Total:	**3**	
3(ii)	Moment of lid about the base = $3 \times (0.2 + 0.1\sin\theta)$	**B1**	θ is the angle the lid makes with the horizontal
	$(6 \times 3 + 2) \times 0.12 = 5 \times 3 \times 0.08 + 2 \times 0.2 + 3 \times (0.2 + 0.1\sin\theta)$	**M1**	Take moments about the base
		A1	
	$\theta = 41.8°$	**A1**	
	Total:	**4**	

Q33

6(i)	$3 \times 0.6 = 8\cos60\,\bar{x}$	**M1**	Takes moments about A
	$\bar{x} = 0.45$ m	**A1**	
	Total:	**2**	
6(ii)	$P\cos60 \times 0.6 = 8 \times 0.45\cos60$	**M1**	Takes moments about A
	$P = 6$ N	**A1**	
	Total:	**2**	

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6(iii)	$\mu = 3\cos30 / (8 - 3\sin30)$	**M1**	Uses $F = \mu R$ used
	$\mu = 6\cos30 / (8 + 6\sin30)$	**M1**	
	$\mu = 0.4$ or 0.472	**A1**	
	$\mu =$ 0.472 accept 0.47	**A1**	
	Total:	**4**	

Q34

Question	Answer	Marks	Guidance
6(i)	$20 \times 3 \times 0.4/8 = 20 \times h/2$	**M1**	Takes moments about the common surface.
	$h = 0.3$ m	**A1**	AG
		2	
6(ii)	Cylinder moment $= 10 \times 0.15/2$	**B1**	
	$20 \times 3 \times 0.4/8 - 10 \times 0.15/2 = 30x$	**M1A1**	Takes moments about the base of the cylinder.
	$x = 0.075$ m	**A1**	
		4	
6(iii)	$30 \times 0.075\sin60 = P \times 0.4\sin60$	**M1A1**	Takes moments about point of contact of the cylinder with the surface.
	$P = 5.625$	**A1**	
		3	

Q35

Question	Answer	Marks	Guidance
2	$0.1\ W + 0.3(60 - W) = 0.25 \times 60$	**M1A1**	Attempts to take moments about the base of the cone. W = weight of the cone.
	$W = 20$ N	**A1**	
		3	

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Q36

Question	Answer	Marks	Guidance
5(i)	$OG = 2 \times 0.8\sin(\pi/4)/(3\pi/4)$ (0.48016…m)	**B1**	
	$AG^2 = (0.8\sin45)^2 + (0.8\cos45 - OG)^2$ OR $AG^2 = 0.8^2 + OG^2 - 2 \times 0.8 \times OG\cos45$	**M1**	Uses Pythagoras's Theorem OR the cosine formula.
	$AG = 0.572(11...)$ m	**A1**	
		3	
5(ii)	$BAG = (0.8\cos45 - OG)/(0.8\sin45)$	**M1**	Uses trigonometry to find angle BAG.
	$BAG = 8.5965° = 8.6(0)°$	**A1**	
	$W \times AG = 12 \times 2 \times 0.8\sin45 \times \sin BAG$	**M1**	Takes moments about A.
	$0.572W = 12 \times 2 \times 0.8\sin45 \times \sin8.6$	**A1FT**	
	$W = 3.55$ N	**A1**	
		5	

Q37

2 (i)	$60(3 \times 0.8/8) \times 0.28 = P(0.8 - 0.8 \times 0.28)$	**M1** **A1**		An attempt at taking moments
	$P = 8.75$ AG	**A1**	3	
(ii)	$\mu = 8.75/60$	**M1**		
	$\mu = 0.146$	**A1**	2	

Q38

4 (i)	$2 \times 0.56 \times 0.28 + 1.2^2(0.56 + 1.2/2) = h(2 \times 0.56 + 1.2^2)$	**M1**		Moments about BC
	$h = 0.775$	**A1**		
	$2 \times 0.56 \times 1 + 1.2^2(1.2/2) = v(2 \times 0.56 + 1.2^2)$	**M1**		Moments about BAG
	$v = 0.775$	**A1**	4	
(ii)	45°	**B1**	1	
(iii)	$\tan\theta = (0.56 + 1.2 - 0.775) / (1.2 - 0.775)$	**M1**		
	$\theta = 66.7°$	**A1**	2	

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Q39

2 (i)	OG = 0.4sin(π/2)/(π/2)	**B1**		= 0.25464...
	d = OG cos70 + 0.4sin70	**M1**		
	d = 0.463 **AG**	**A1**	3	
(ii)	0.463W = 15 × 0.8cos35	**M1**		
	W = 21.2 N	**A1**	2	

Q40

4 (i)	μ = Wsin20/(Wcos20)	**M1**		μ = tan20
	μ = 0.364	**A1**	2	
(ii)	Wx/2 + W(x+4.4/4) = 2WOG	**M1** **A1**		Attempts to take moments OG = distance to C from M
	OG = 0.4tan70 (= 0.4/tan20)	**B1**		
	x = 0.732	**A1**	4	

Q41

4 (i)	(0.7 × 0.4) × 0.2= (0.28 − 0.03) x + 0.03 × 0.1	**M1**		
	x = 0.212 **AG**	**A1**		
	(0.7 × 0.4) × 0.35 = (0.28 − 0.03)y + 0.03 × 0.2	**M1**		
	y = 0.368	**A1**	4	
(ii)	0.4F = 0.212 × 70	**M1**		Topples about A
	F = 37.1	**A1**	2	
(iii)	θ = tan^{-1} [(0.4-0.212)/0.368]	**M1**		
	θ = 27.1	**A1**	2	

Q42

2 (i)	CoM(large) = 0.6/(π/2) or CoM(small) = 0.3/(π/2)	**B1**		
	(π x 0.6 + π x 0.3)D = π x 0.6(1.2/π) – π x 0.3(0.6/π)	**M1**		OR (2+1)D = 2(1.2/π) – 1(0.6/π) Moments about ACB
	D = 0.191 m AG	**A1**	3	
(ii)	(π x 0.6 + π x 0.3)H = π x 0.6 x 0.6 + π x 0.3 x 0.9	**M1**		OR 3H = 2 x 0.6 + 1 x 0.9 Moments about A
	H = 0.7	**A1**		
	tanθ = 0.191/0.7	**M1**		
	θ = 15.3°	**A1**	4	

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Q43

4 (i)	$(0.9a + 0.9a/2)Y =$ $0.9a$ x 0.45 + 0.45a x 0.9 x 2/3	**M1**		$1.5Y = 1$ x 0.45 + 0.5 x 0.6 Moments about AD
	$Y = 0.5$ m	**A1**	2	
(ii)	$(0.9a + 0.9a/2)X =$ $0.9a$ x $a/2$ + 0.45a x (a + a/3)	**M1**		$1.5X = 1$ x $a/2$ + 0.5 x 4a/3
	$X = 7a/9$	**A1**	2	
(iii)	0.5 x 6 = (a – 7a/9) x 18	**M1** **A1**√		Ft [Y**i** and (a–X**ii**)]
	$a = 0.75$	**A1**	3	

Q44

3 (i)	Fcos45 = 10cos60	**M1**		Resolving horizontally
	$F = 7.07$	**A1**		7.071 ..= 5√2
	Fsin45 + 10sin60 = W	**M1**		Resolving vertically
	$W = 13.7$	**A1**	[4]	13.660.. = 5(√2+√3)
(ii)		**M1**		Moments about A
	Wdcos30 = (Fsin75)0.5	**A1**		
	$d = 0.289$ m	**A1**	[3]	

Q45

6 (i)	CoM semi-circle from $DF = 4r/3\pi$	**B1**		
	(0.4 x 1.8) x 0.2 = (πr^2 /2) x (4r/3π)	**M1**		Moments about A
	$r = 0.6$ AG	**A1**	[3]	
(ii)	Pcos60(0.4 + 0.6cos60)	**B1**		Moment of vertical component
	Pcos30(1.8 – 0.6 + 0.6sin60)	**B1**		Moment of horiz component
	15 x 0.4 =	**M1**		
	Pcos60(0.4 + 0.6cos60) + Pcos30(1.8 –0.6 + 0.6sin60) $P = 3.26$ N AG	**A1**	[4]	3.2622...
(iii)	μ =3.262sin60/(15 – 3.262cos60)	**M1**		
	$\mu = 0.211$	**A1**	[2]	

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Circular Motion

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Q1

2 A hollow hemispherical bowl of radius a has a smooth inner surface and is fixed with its axis vertical. A particle P of mass m moves in horizontal circles on the inner surface of the bowl, at a height x above the lowest point of the bowl. The speed of P is $\sqrt{\frac{8}{3}ga}$.

Find x in terms of a. [6]

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Q2

5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle completes vertical circles with centre O. The points A and B are on the path of P, both on the same side of the vertical through O. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O.

The speed of P when it is at A is u and the speed of P when it is at B is $\sqrt{ag}$. The tensions in the string at A and B are T_A and T_B respectively. It is given that $T_A = 7T_B$.

Find the value of θ and find an expression for u in terms of a and g. [8]

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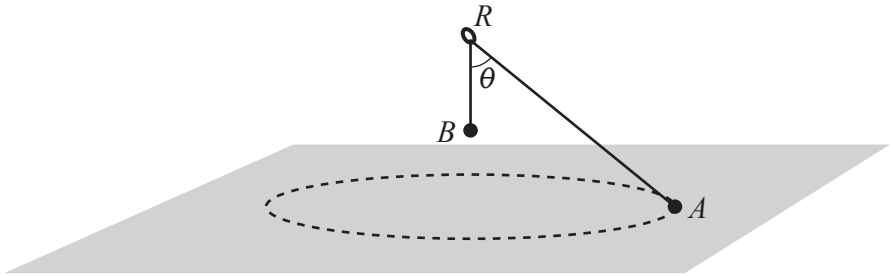
Q3

3



Particles A and B, of masses $3m$ and m respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles on a smooth horizontal surface with speed $\frac{2}{5}\sqrt{ga}$. The angle between AR and BR is θ (see diagram). The normal reaction between A and the surface is $\frac{12}{5}mg$.

(a) Find $\cos\theta$. [3]

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(b) Find, in terms of a, the distance of B below the ring. [3]

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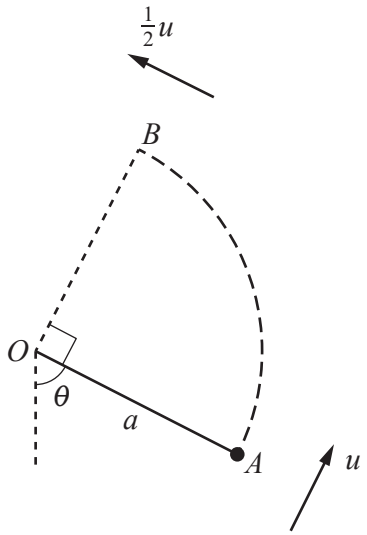
Q4

4



A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is initially held with the string taut at the point A, where OA makes an angle θ with the downward vertical through O. The particle is then projected with speed u perpendicular to OA and begins to move upwards in part of a vertical circle. The string goes slack when the particle is at the point B where angle AOB is a right angle. The speed of the particle when it is at B is $\frac{1}{2}u$ (see diagram).

Find the tension in the string at A, giving your answer in terms of m and g. [8]

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Q5

1 One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O on a smooth horizontal plane. A particle P of mass m is attached to the other end of the string and moves in a horizontal circle with centre O. The speed of P is $\sqrt{\frac{4}{3}ga}$.

Find the extension of the string. [4]

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Q6

6 A particle P, of mass m, is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P moves in complete vertical circles about O with the string taut. The points A and B are on the path of P with AB a diameter of the circle. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O. The speed of P when it is at A is $\sqrt{5ag}$.

The ratio of the tension in the string when P is at A to the tension in the string when P is at B is 9 : 5.

(a) Find the value of $\cos\theta$. [6]

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(b) Find, in terms of a and g, the greatest speed of P during its motion. [2]

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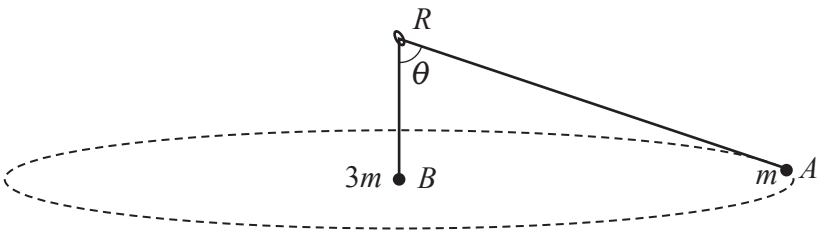
3



Particles A and B, of masses m and $3m$ respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles with speed v. Particles A and B are at the same horizontal level. The angle between AR and BR is θ (see diagram).

(a) Show that $\cos\theta = \frac{1}{3}$. [2]

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(b) Find an expression for v in terms of a and g. [4]

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Q8

7 One end of a light inextensible string of length a is attached to a fixed point O. The other end of the string is attached to a particle P of mass m. The particle P is held vertically below O with the string taut and then projected horizontally. When the string makes an angle of 60° with the upward vertical, P becomes detached from the string. In its subsequent motion, P passes through the point A which is a distance a vertically above O.

(a) The speed of P when it becomes detached from the string is V. Use the equation of the trajectory of a projectile to find V in terms of a and g. [4]

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(b) Find, in terms of m and g, the tension in the string immediately after P is initially projected horizontally. [4]

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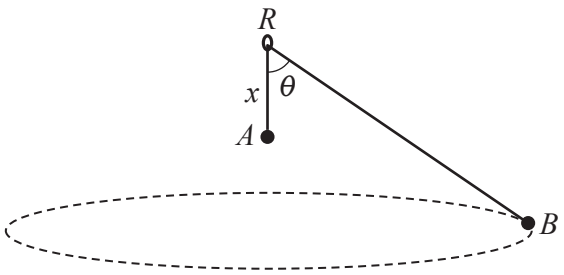
Q9

2



A light inextensible string of length a is threaded through a fixed smooth ring R. One end of the string is attached to a particle A of mass $3m$. The other end of the string is attached to a particle B of mass m. The particle A hangs in equilibrium at a distance x vertically below the ring. The angle between AR and BR is θ (see diagram). The particle B moves in a horizontal circle with constant angular speed $2\sqrt{\frac{g}{a}}$.

Show that $\cos\theta = \frac{1}{3}$ and find x in terms of a. [5]

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Q10

7 A hollow cylinder of radius a is fixed with its axis horizontal. A particle P, of mass m, moves in part of a vertical circle of radius a and centre O on the smooth inner surface of the cylinder. The speed of P when it is at the lowest point A of its motion is $\sqrt{\frac{7}{2}ga}$.

The particle P loses contact with the surface of the cylinder when OP makes an angle θ with the upward vertical through O.

(a) Show that $\theta = 60°$. [5]

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(b) Show that in its subsequent motion P strikes the cylinder at the point A. [5]

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Q11

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle P moves in horizontal circles about O. The tension in the string is $4mg$.

Find, in terms of a and g, the time that P takes to make one complete revolution. [2]

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3 A particle Q of mass m is attached to a fixed point O by a light inextensible string of length a. The particle moves in complete vertical circles about O. The points A and B are on the path of Q with AB a diameter of the circle. OA makes an angle of 60° with the downward vertical through O and OB makes an angle of 60° with the upward vertical through O. The speed of Q when it is at A is $2\sqrt{ag}$.

Given that T_A and T_B are the tensions in the string at A and B respectively, find the ratio $T_A : T_B$. [6]

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Q13

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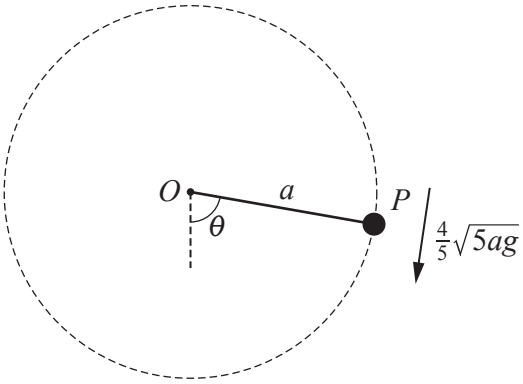

A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held with the string taut and making an angle θ with the downward vertical. The particle P is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

Find the value of $\cos\theta$. [5]

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Q14

3 One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O. The length of the string during this motion is $(k+1)a$.

(a) Find the value of k. [4]

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(b) Find the value of $\cos\theta$. [2]

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Q15

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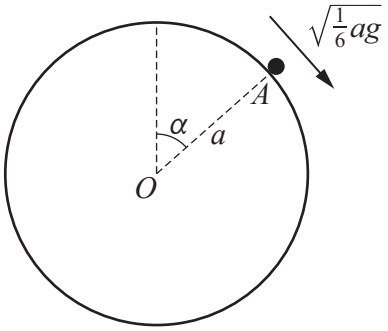

A fixed smooth solid sphere has centre O and radius a. A particle of mass m is projected downwards with speed $\sqrt{\frac{1}{6}ag}$ from the point A on the surface of the sphere, where OA makes an angle α with the upward vertical through O (see diagram). The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point B, where OB makes an angle β with the upward vertical through O.

Given that $\cos\alpha = \frac{2}{3}$, find the value of $\cos\beta$. [5]

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Q16

4 A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r. The angle between the vertical and the normal reaction of the surface on P is θ.

(a) Show that $\cos\theta = \dfrac{g}{\omega^2 r}$. [3]

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The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

(b) Find x in terms of r. [4]

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Q17

11 Answer only **one** of the following two alternatives.

EITHER

A particle P, of mass m, is able to move in a vertical circle on the smooth inner surface of a sphere with centre O and radius a. Points A and B are on the inner surface of the sphere and AOB is a horizontal diameter. Initially, P is projected vertically downwards with speed $\sqrt{\left(\frac{21}{2}ag\right)}$ from A and begins to move in a vertical circle. At the lowest point of its path, vertically below O, the particle P collides with a stationary particle Q, of mass $4m$, and rebounds. The speed acquired by Q, as a result of the collision, is just sufficient for it to reach the point B.

(i) Find the speed of P and the speed of Q immediately after their collision. [7]

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In its subsequent motion, P loses contact with the inner surface of the sphere at the point D, where the angle between OD and the upward vertical through O is θ.

(ii) Find $\cos\theta$. [5]

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Q18

2 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is moving in a complete vertical circle about O. The points A and B are on the circle, at opposite ends of a diameter, and such that OA makes an acute angle α with the upward vertical through O. The speed of P as it passes through A is $\frac{3}{2}\sqrt{(ag)}$. The tension in the string when P is at B is four times the tension in the string when P is at A.

(i) Show that $\cos\alpha = \frac{3}{4}$. [6]

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(ii) Find the tension in the string when P is at B. [2]

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Q19

4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O and P is held with the string taut and horizontal. The particle P is projected vertically downwards with speed $\sqrt{(2ag)}$ so that it begins to move along a circular path. The string becomes slack when OP makes an angle θ with the upward vertical through O.

(i) Show that $\cos\theta = \frac{2}{3}$. [5]

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(ii) Find the greatest height, above the horizontal through O, reached by P in its subsequent motion. [4]

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Q20

11 Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held so that the string is taut, with OP horizontal. The particle is projected downwards with speed $\sqrt{\left(\frac{2}{5}ag\right)}$ and begins to move in a vertical circle. The string breaks when its tension is equal to $\frac{11}{5}mg$.

(i) Show that the string breaks when OP makes an angle θ with the downward vertical through O, where $\cos\theta = \frac{3}{5}$. Find the speed of P at this instant. [6]

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(ii) For the subsequent motion after the string breaks, find the distance OP when the particle P is vertically below O. [6]

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Q21

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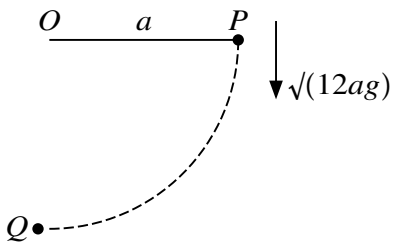

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held with the string taut and horizontal. It is projected downwards with speed $\sqrt{(12ag)}$. At the lowest point of its motion, P collides directly with a particle Q of mass km which is at rest (see diagram). In the collision, P and Q coalesce. The tension in the string immediately after the collision is half of its value immediately before the collision. Find the possible values of k. [11]

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Q22

11 Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a. The centre of the sphere is O and the point C is on the inner surface of the sphere, vertically below O. The points A and B on the inner surface of the sphere are the ends of a diameter of the sphere. The diameter AOB makes an acute angle α with the vertical, where $\cos\alpha = \frac{4}{5}$, with A below the horizontal level of B. The particle is projected from A with speed u, and moves along the inner surface of the sphere towards C. The normal reaction forces on the particle at A and C are in the ratio 8 : 9.

(i) Show that $u^2 = 4ag$. [6]

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(ii) Determine whether P reaches B without losing contact with the inner surface of the sphere. [6]

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Q23

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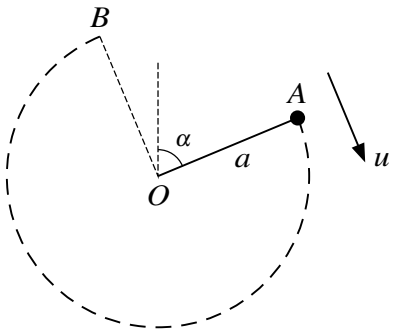

A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The point A is such that $OA = a$ and OA makes an angle α with the upward vertical, where $\tan\alpha = \frac{12}{5}$. The particle is projected downwards from A with speed u perpendicular to the string and moves in a vertical plane (see diagram). The string becomes slack after the string has rotated through 270° from its initial position, with the particle now at the point B.

(i) Show that $u^2 = 2ag$. [5]

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(ii) Find the maximum tension in the string as the particle moves from A to B. [4]

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Q24

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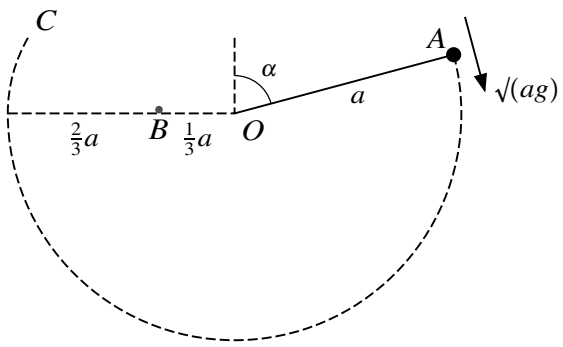

A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The point A is such that $OA = a$ and OA makes an angle α with the upward vertical through O. The particle is held at A and then projected downwards with speed $\sqrt{(ag)}$ so that it begins to move in a vertical circle with centre O. There is a small smooth peg at the point B which is at the same horizontal level as O and at a distance $\frac{1}{3}a$ from O on the opposite side of O to A (see diagram).

(i) Show that, when the string first makes contact with the peg, the speed of the particle is $\sqrt{\big(ag(1 + 2\cos\alpha)\big)}$. [2]

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The particle now begins to move in a vertical circle with centre B. When the particle is at the point C where angle $CBO = 150°$, the tension in the string is the same as it was when the particle was at the point A.

(ii) Find the value of $\cos\alpha$. [10]

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Q25

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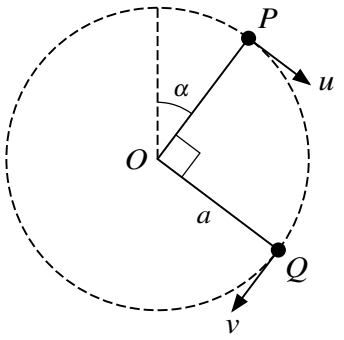

A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is moving in complete vertical circles with the string taut. When the particle is at the point P, where OP makes an angle α with the upward vertical through O, its speed is u. When the particle is at the point Q, where angle $QOP = 90°$, its speed is v (see diagram). It is given that $\cos\alpha = \frac{4}{5}$.

(i) Show that $v^2 = u^2 + \frac{14}{5}ag$. [2]

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The tension in the string when the particle is at Q is twice the tension in the string when the particle is at P.

(ii) Obtain another equation relating u^2, v^2, a and g, and hence find u in terms of a and g. [5]

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(iii) Find the least tension in the string during the motion. [3]

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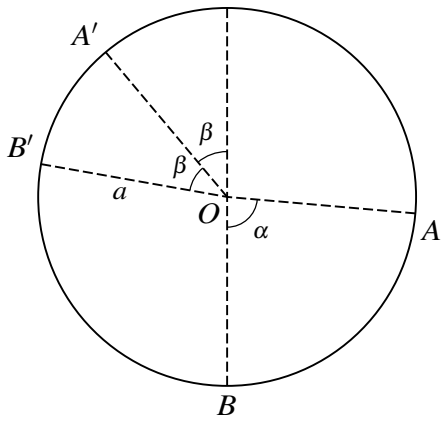
Q26

11 Answer only **one** of the following two alternatives.

EITHER



A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a. The centre of the sphere is O. The points A and A' are on the inner surface of the sphere, on opposite sides of the vertical through O; the radius OA makes an angle α with the downward vertical and the radius OA' makes an angle β with the upward vertical. The point B is on the inner surface of the sphere, vertically below O. The point B' is on the inner surface of the sphere and such that OB' makes an angle 2β with the upward vertical through O (see diagram). It is given that $\cos\alpha = \frac{1}{16}$.

(i) P is projected from A with speed u along the surface of the sphere downwards towards B. Subsequently it loses contact with the sphere at A'. Show that $u^2 = \frac{1}{8}ag(1 + 24\cos\beta)$. [5]

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(ii) P is now projected from B with speed u along the surface of the sphere towards B'. Subsequently it loses contact with the sphere at B'. Find $\cos\beta$. [6]

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(iii) In part **(i)**, the reaction of the sphere on P when it is initially projected at A is R. Find R in terms of m and g. [3]

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4 A particle P is at rest at the lowest point on the smooth inner surface of a hollow sphere with centre O and radius a. The particle is projected horizontally with speed u and begins to move in a vertical circle on the inner surface of the sphere. The particle loses contact with the sphere at the point A, where OA makes an angle θ with the upward vertical through O. Given that the speed of P at A is $\sqrt{(\frac{3}{5}ag)}$, find u in terms of a and g. [5]

Find, in terms of a, the greatest height above the level of O achieved by P in its subsequent motion. (You may assume that P achieves its greatest height before it makes any further contact with the sphere.) [5]

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1 A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle moves in complete vertical circles with centre O. The tension in the string when P is at its lowest point is twice the tension in the string when P is at its highest point. Find, in terms of a and g, the greatest speed of P during the motion. [6]

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4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held vertically above O with the string taut and then projected horizontally with speed $\sqrt{\left(\frac{13}{3}ag\right)}$. It begins to move in a vertical circle with centre O. When P is at its lowest point, it collides with a stationary particle of mass λm. The two particles coalesce.

(i) Show that the speed of the combined particle immediately after the impact is $\frac{5}{\lambda+1}\sqrt{\left(\frac{1}{3}ag\right)}$. [4]

In the subsequent motion, the string becomes slack when the combined particle is at a height of $\frac{1}{3}a$ above the level of O.

(ii) Find the value of λ. [6]

(iii) Find, in terms of m and g, the instantaneous change in the tension in the string as a result of the collision. [4]

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3 A particle P, of mass m, is placed at the highest point of a fixed solid smooth sphere with centre O and radius a. The particle P is given a horizontal speed u and it moves in part of a vertical circle, with centre O, on the surface of the sphere. When OP makes an angle θ with the upward vertical, and P is still in contact with the surface of the sphere, the speed of P is v and the reaction of the sphere on P has magnitude R. Show that $R = mg(3\cos\theta - 2) - \dfrac{mu^2}{a}$. [5]

The particle loses contact with the sphere at the instant when $v = 2u$. Find u in terms of a and g. [4]

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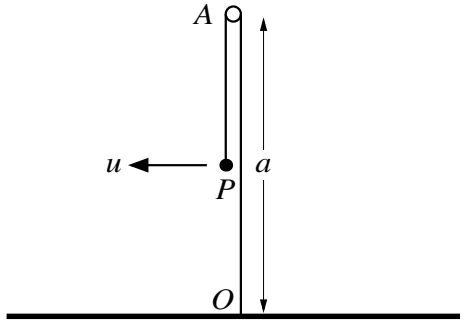
10 Answer only **one** of the following two alternatives.

EITHER



One end of a light inextensible string of length $\frac{3}{2}a$ is attached to a fixed point O on a horizontal surface. The other end of the string is attached to a particle P of mass m. The string passes over a small fixed smooth peg A which is at a distance a vertically above O. The system is in equilibrium with P hanging vertically below A and the string taut. The particle is projected horizontally with speed u (see diagram). When P is at the same horizontal level as A, the tension in the string is T. Show that $T = \frac{2m}{a}(u^2 - ag)$. [3]

The ratio of the tensions in the string immediately before, and immediately after, the string loses contact with the peg is 5 : 1.

(i) Show that $u^2 = 5ag$. [6]

(ii) Find, in terms of m and g, the tension in the string when P is next at the same horizontal level as A. [5]

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4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. When P is hanging at rest vertically below O, it is projected horizontally. In the subsequent motion P completes a vertical circle. The speed of P when it is at its highest point is u. Show that the least possible value of u is $\sqrt{(ag)}$. [2]

It is now given that $u = \sqrt{(ag)}$. When P passes through the lowest point of its path, it collides with, and coalesces with, a stationary particle of mass $\frac{1}{4}m$. Find the speed of the combined particle immediately after the collision. [4]

In the subsequent motion, when OP makes an angle θ with the upward vertical the tension in the string is T. Find an expression for T in terms of m, g and θ. [5]

Find the value of $\cos\theta$ when the string becomes slack. [2]

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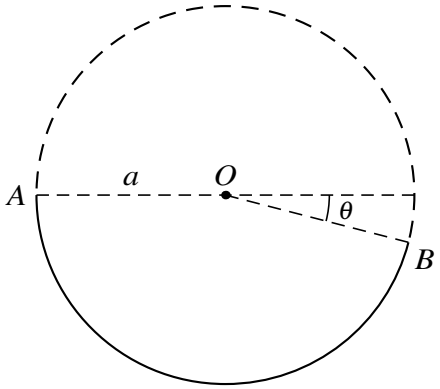
Q33

4



A smooth wire is in the form of an arc AB of a circle, of radius a, that subtends an obtuse angle $\pi - \theta$ at the centre O of the circle. It is given that $\sin\theta = \frac{1}{4}$. The wire is fixed in a vertical plane, with AO horizontal and B below the level of O (see diagram). A small bead of mass m is threaded on the wire and projected vertically downwards from A with speed $\sqrt{(\frac{3}{10}ga)}$.

(i) Find the reaction between the bead and the wire when the bead is vertically below O. [3]

(ii) Find the speed of the bead as it leaves the wire at B. [3]

(iii) Show that the greatest height reached by the bead is $\frac{1}{8}a$ above the level of O. [4]

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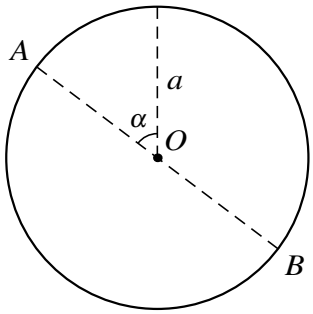

A smooth cylinder of radius a is fixed with its axis horizontal. The point O is the centre of a circular cross-section of the cylinder. The line AOB is a diameter of this circular cross-section and the radius OA makes an angle α with the upward vertical (see diagram). It is given that $\cos\alpha = \frac{3}{5}$. A particle P of mass m moves on the inner surface of the cylinder in the plane of the cross-section. The particle passes through A with speed u along the surface in the downwards direction. The magnitude of the reaction between P and the inner surface of the sphere is R_A when P is at A, and is R_B when P is at B. It is given that $R_B = 10R_A$. Show that $u^2 = ag$. [6]

The particle loses contact with the surface of the cylinder when OP makes an angle θ with the upward vertical. Find the value of $\cos\theta$. [4]

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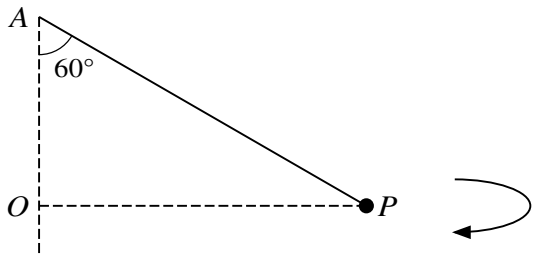
Q35

4



A particle P of mass 0.3 kg is attached to a fixed point A by a light elastic string of natural length 0.8 m and modulus of elasticity 16 N. The particle P moves in a horizontal circle which has centre O. It is given that AO is vertical and that angle OAP is 60° (see diagram). Calculate the speed of P. [6]

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Q36

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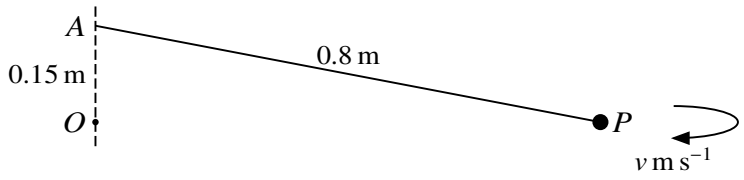

A particle P of mass 0.3 kg is attached to a fixed point A by a light inextensible string of length 0.8 m. The fixed point O is 0.15 m vertically below A. The particle P moves with constant speed v m s^{-1} in a horizontal circle with centre O (see diagram).

(i) Show that the tension in the string is 16 N. [2]

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(ii) Find the value of v. [3]

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Q37

3 A particle P of mass 0.4 kg is attached to a fixed point A by a light inextensible string of length 0.5 m. The point A is 0.3 m above a smooth horizontal surface. The particle P moves in a horizontal circle on the surface with constant angular speed 5 rad s^{-1}.

(i) Calculate the tension in the string. [3]

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(ii) Find the magnitude of the force exerted by the surface on P. [2]

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Q38

6 A and B are two fixed points on a vertical axis with A 0.6 m above B. A particle P of mass 0.3 kg is attached to A by a light inextensible string of length 0.5 m. The particle P is attached to B by a light elastic string with modulus of elasticity 46 N. The particle P moves with constant angular speed $8\,\text{rad}\,\text{s}^{-1}$ in a horizontal circle with centre at the mid-point of AB.

(i) Find the speed of P. [2]

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(ii) Calculate the tension in the string BP and hence find the natural length of this string. [7]

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Q39

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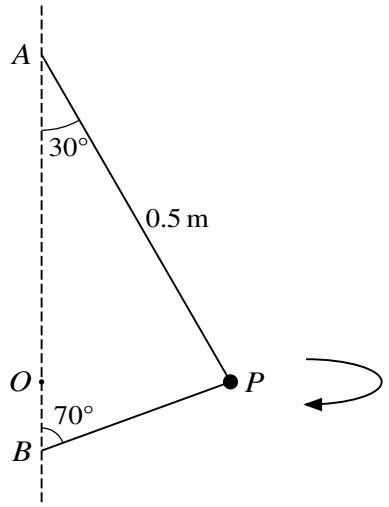

A and B are two fixed points on a vertical axis with A above B. A particle P of mass 0.4 kg is attached to A by a light inextensible string of length 0.5 m. The particle P is attached to B by another light inextensible string. P moves with constant speed in a horizontal circle with centre O between A and B. Angle $BAP = 30°$ and angle $ABP = 70°$ (see diagram).

(i) Given that the tensions in the two strings are equal, find the speed of P. [5]

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(ii) Given instead that the angular speed of P is $12\,\text{rad}\,\text{s}^{-1}$, find the tensions in the strings. [5]

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Q40

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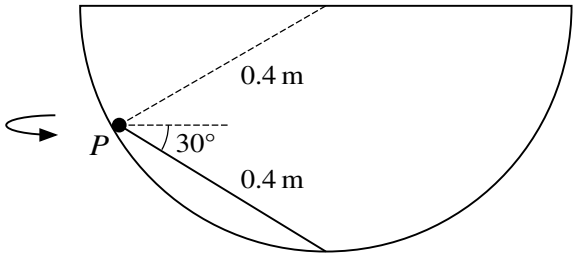

One end of a light inextensible string of length 0.4 m is attached to the lowest point of a hemisphere of radius 0.4 m fixed with its axis vertical. A particle P of mass 0.3 kg is attached to the other end of the string. The string is straight and makes an angle of 30° with the horizontal. P moves on the smooth inner surface of the hemisphere in a horizontal circle (see diagram).

(i) Calculate the smallest possible angular speed of P. [4]

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(ii) Given that the greatest possible tension in the string is 5 N, calculate the greatest possible speed of P. [4]

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Q41

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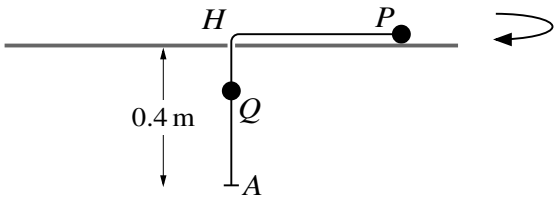

A particle P of mass 0.2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a particle Q of mass 0.3 kg. The string passes through a small hole H in a smooth horizontal surface. A light elastic string of natural length 0.3 m and modulus of elasticity 15 N joins Q to a fixed point A which is 0.4 m vertically below H. The particle P moves on the surface in a horizontal circle with centre H (see diagram).

(i) Calculate the greatest possible speed of P for which the elastic string is not extended. [4]

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(ii) Find the distance HP given that the angular speed of P is $8\,\text{rad}\,\text{s}^{-1}$. [5]

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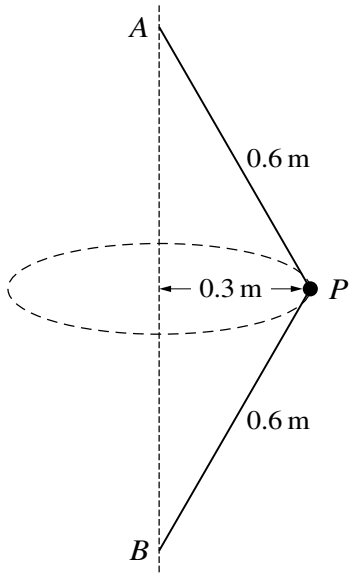

A particle P of mass 0.2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point A. The particle P is also attached to one end of a second light inextensible string of length 0.6 m, the other end of which is attached to a fixed point B vertically below A. The particle moves in a horizontal circle of radius 0.3 m, which has its centre at the mid-point of AB, with both strings straight (see diagram).

(i) Calculate the least possible angular speed of P. [4]

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The string AP will break if its tension exceeds 8 N. The string BP will break if its tension exceeds 5 N.

(ii) Find the greatest possible speed of P. [5]

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Q43

5 A particle P of mass 0.1 kg is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point A. The particle P moves in a circle which has its centre O on a smooth horizontal surface 0.3 m below A. The tension in the string has magnitude T N and the magnitude of the force exerted on P by the surface is R N.

(i) Given that the speed of P is $1.5\,\text{m}\,\text{s}^{-1}$, calculate T and R. [4]

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(ii) Given instead that $T = R$, calculate the angular speed of P. [4]

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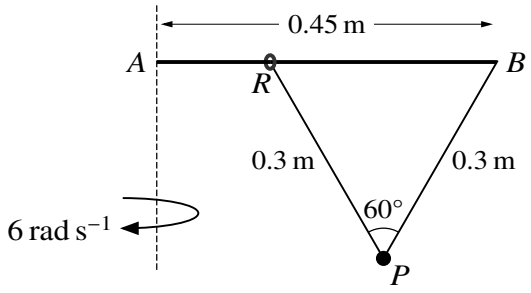

A rough horizontal rod AB of length 0.45 m rotates with constant angular velocity 6 rad s^{-1} about a vertical axis through A. A small ring R of mass 0.2 kg can slide on the rod. A particle P of mass 0.1 kg is attached to the mid-point of a light inextensible string of length 0.6 m. One end of the string is attached to R and the other end of the string is attached to B, with angle $RPB = 60°$ (see diagram). R and P move in horizontal circles as the system rotates. R is in limiting equilibrium.

(i) Show that the tension in the portion PR of the string is 1.66 N, correct to 3 significant figures. [5]

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(ii) Find the coefficient of friction between the ring and the rod. [5]

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Q45

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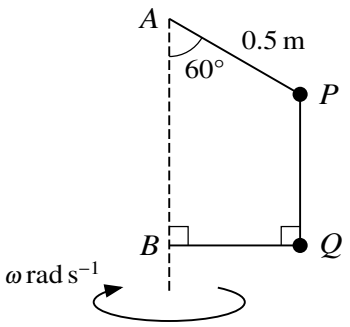

Two particles P and Q have masses 0.4 kg and m kg respectively. P is attached to a fixed point A by a light inextensible string of length 0.5 m which is inclined at an angle of 60° to the vertical. P and Q are joined to each other by a light inextensible vertical string. Q is attached to a fixed point B, which is vertically below A, by a light inextensible string. The string BQ is taut and horizontal. The particles rotate in horizontal circles about an axis through A and B with constant angular speed ω rad s^{-1} (see diagram). The tension in the string joining P and Q is 1.5 N.

(i) Find the tension in the string AP and the value of ω. [4]

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(ii) Find m and the tension in the string BQ. [3]

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Q46

6 A particle P of mass 0.15 kg is attached to one end of a light elastic string of natural length 0.4 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point A. The particle P moves in a horizontal circle which has its centre vertically below A, with the string inclined at $\theta°$ to the vertical and $AP = 0.5$ m.

(i) Find the angular speed of P and the value of θ. [5]

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(ii) Calculate the difference between the elastic potential energy stored in the string and the kinetic energy of P. [4]

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Q47

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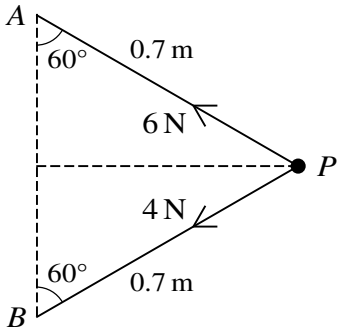

The ends of two light inextensible strings of length 0.7 m are attached to a particle P. The other ends of the strings are attached to two fixed points A and B which lie in the same vertical line with A above B. The particle P moves in a horizontal circle which has its centre at the mid-point of AB. Both strings are inclined at 60° to the vertical. The tension in the string attached to A is 6 N and the tension in the string attached to B is 4 N (see diagram).

(i) Find the mass of P. [2]

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(ii) Calculate the speed of P. [3]

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Q48

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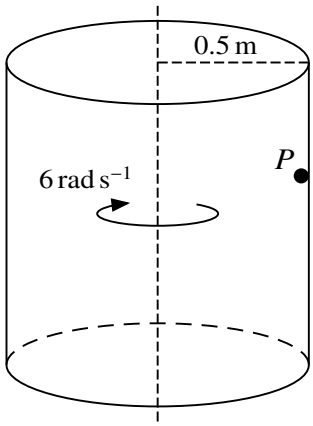

A hollow cylinder with a rough inner surface has radius 0.5 m. A particle P of mass 0.4 kg is in contact with the inner surface of the cylinder. The particle and cylinder rotate together with angular speed 6 rad s^{-1} about the vertical axis of the cylinder, so that the particle moves in a horizontal circle (see diagram). Given that P is about to slip downwards, find the coefficient of friction between P and the surface of the cylinder. [4]

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Q49

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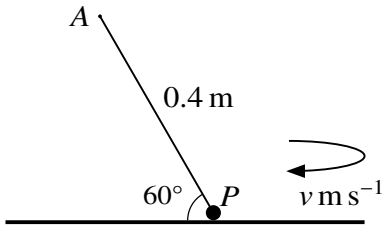

One end of a light inextensible string of length 0.4 m is attached to a fixed point A which is above a smooth horizontal surface. A particle P of mass 0.6 kg is attached to the other end of the string. P moves in a circle on the surface with constant speed v m s^{-1}, with the string taut and making an angle of 60° with the horizontal (see diagram).

(i) Given that $v = 0.5$, calculate the magnitude of the force that the surface exerts on P. [4]

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(ii) Find the greatest possible value of v for which P remains in contact with the surface. [3]

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Q50

6 One end of a light elastic string of natural length 0.4 m and modulus of elasticity 8 N is attached to a fixed point O on a smooth horizontal plane. The other end of the string is attached to a particle P of mass 0.2 kg which moves on the plane in a circular path with centre O. The speed of P is v m s^{-1} and the extension of the string is x m.

(i) Given that $v = 2.5$, find x. [4]

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It is given instead that the kinetic energy of P is twice the elastic potential energy stored in the string.

(ii) Form two simultaneous equations and hence find x and v. [5]

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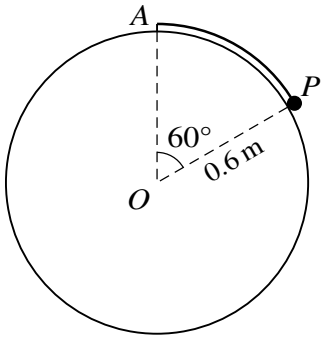
Q51

7



One end of a light inextensible string is attached to the highest point A of a solid fixed sphere with centre O and radius 0.6 m. The other end of the string is attached to a particle P of mass 0.2 kg which rests in contact with the smooth surface of the sphere. The angle $AOP = 60°$ (see diagram). The sphere exerts a contact force of magnitude R N on P and the tension in the string is T N.

(i) By resolving vertically, show that $R + (\sqrt{3})T = 4$. [2]

P is now set in motion, and moves with angular speed ω rad s^{-1} in a horizontal circle on the surface of the sphere.

(ii) Find an equation involving R, T and ω. [2]

(iii) Hence

(a) calculate R when $\omega = 2$, [2]

(b) find the greatest possible value of T and the corresponding speed of P. [4]

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Q52

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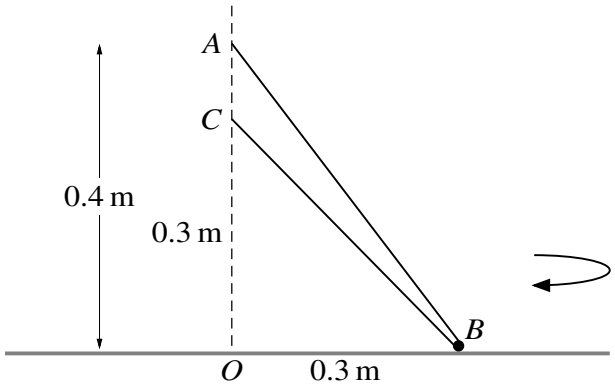

A light inextensible string passes through a small smooth bead B of mass 0.4 kg. One end of the string is attached to a fixed point A 0.4 m above a fixed point O on a smooth horizontal surface. The other end of the string is attached to a fixed point C which is vertically below A and 0.3 m above the surface. The bead moves with constant speed on the surface in a circle with centre O and radius 0.3 m (see diagram).

(i) Given that the tension in the string is 2 N, calculate

(a) the angular speed of the bead, [3]

(b) the magnitude of the contact force exerted on the bead by the surface. [2]

(ii) Given instead that the bead is about to lose contact with the surface, calculate the speed of the bead. [4]

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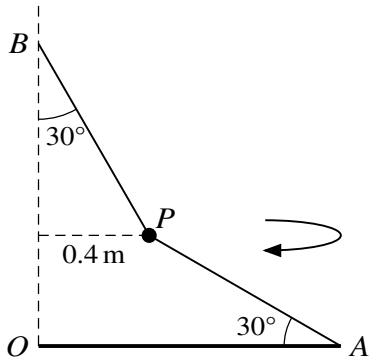
[Turn over

6



OA is a rod which rotates in a horizontal circle about a vertical axis through *O*. A particle *P* of mass 0.2 kg is attached to the mid-point of a light inextensible string. One end of the string is attached to the rod at *A* and the other end of the string is attached to a point *B* on the axis. It is given that *OA* = *OB*, angle *OAP* = angle *OBP* = 30°, and *P* is 0.4 m from the axis. The rod and the particle rotate together about the axis with *P* in the plane *OAB* (see diagram).

(i) Calculate the tensions in the two parts of the string when the speed of *P* is $1.2\,\text{m}\,\text{s}^{-1}$. [6]

The angular speed of the rod is increased to $5\,\text{rad}\,\text{s}^{-1}$, and it is given that the system now rotates with angle *OAP* = angle *OBP* = 60°.

(ii) Show that the tension in the part *AP* of the string is zero. [6]

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Q54

1 A particle P of mass 0.3 kg moves in a circle with centre O on a smooth horizontal surface. P is attached to O by a light elastic string of modulus of elasticity 12 N and natural length l m. The speed of P is 4 m s^{-1}, and the radius of the circle in which it moves is $2l$ m. Calculate l. [4]

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5 A small ball B of mass 0.4 kg moves in a horizontal circle with centre O and radius 0.6 m on a smooth horizontal surface. One end of a light inextensible string is attached to B; the other end of the string is attached to a fixed point 0.45 m vertically above O.

(i) Given that the tension in the string is 5 N, calculate the speed of B. [3]

(ii) Find the greatest possible tension in the string for the motion, and the corresponding angular speed of B. [4]

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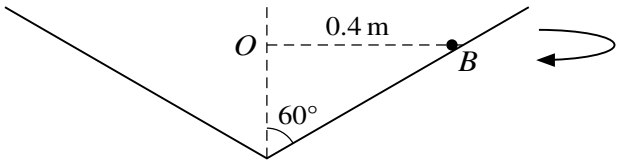
Q56

7



A small ball B of mass 0.5 kg moves in a horizontal circle with centre O and radius 0.4 m on the smooth inner surface of a hollow cone fixed with its vertex down. The axis of the cone is vertical and the semi-vertical angle is 60° (see diagram).

(i) Show that the magnitude of the force exerted by the cone on B is 5.77 N, correct to 3 significant figures, and calculate the angular speed of B. [4]

One end of a light elastic string of natural length 0.45 m and modulus of elasticity 36 N is attached to B. The other end of the string is attached to the point on the axis 0.3 m above O. The ball B again moves on the surface of the cone in the same horizontal circle as before.

(ii) Calculate the speed of B. [6]

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Circular Motion MS

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Q1

Question	Answer	Marks	Guidance
2	$\uparrow R\cos\theta = mg$	**B1**	
	$\rightarrow R\sin\theta = \frac{mv^2}{r}$	**B1**	
	$r = a\sin\theta$	**B1**	
	$8\cos\theta = 3\left(1-(\cos\theta)^2\right)$	**M1**	Quadratic equation in $\cos\theta$.
	$\cos\theta = \frac{1}{3}$	**A1**	
	$x = \frac{2}{3}a$	**A1**	
		6	

Q2

Question	Answer	Marks	Guidance
5	$T_A - mg\cos\theta = \frac{mu^2}{a}$	**B1**	
	$T_B + mg\cos\theta = \frac{mag}{a}$	**B1**	
	$T_A = 7T_B$ so $mg\cos\theta + \frac{mu^2}{a} = 7\left(-mg\cos\theta + \frac{mag}{a}\right)$ $u^2 = ag(7-8\cos\theta)$	**M1**	Use given relationship and combine.
	Energy: $\frac{1}{2}mu^2 - \frac{1}{2}mag = mg(a\cos\theta + a\cos\theta)$ So $u^2 = ag(4\cos\theta+1)$	**M1 A1**	Energy equation.
	Equate expressions for u^2	**M1**	
	$\cos\theta = \frac{1}{2}$, $\theta = 60°$	**A1**	CAO
	$u = \sqrt{3ga}$	**A1**	CAO
		8	

Q3

Question	Answer	Marks	Guidance
3(a)	For B: $T = mg$	**B1**	May be embedded.
	For A: $R + T\cos\theta = 3mg$	**M1**	All 3 terms, allow sign errors, allow sin/cos mix.
	Use given R to obtain $\cos\theta = \frac{3}{5}$	**A1**	
		3	
3(b)	$T\sin\theta = \frac{3mv^2}{r}$	**M1**	May be seen in part **(a)**, allow sin/cos mix.
	$r = AR\sin\theta$	**B1**	Or equivalent.
	[Combine to give $AR = \frac{3a}{4}$, so] $BR = \frac{1}{4}a$	**A1**	
		3	

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Q4

Question	Answer	Marks	Guidance
4	$\frac{1}{2}mu^2-\frac{1}{2}m\left(\frac{u}{2}\right)^2=mg\left(a\cos\theta+a\sin\theta\right)$	*M1	Energy equation, with 2 KE terms and a two-part GPE term, allow cos/sin mix.
	$\frac{3}{4}u^2=2ag\left(\cos\theta+\sin\theta\right)$	A1	
	At B, tension in string is zero, so $mg\sin\theta=\frac{m\left(\frac{u}{2}\right)^2}{a}$ $(u^2=4ag\sin\theta)$	B1	N2L
	Eliminate u^2	DM1	
	$\tan\theta=2$ OE	A1	
	At A, $T-mg\cos\theta=\frac{mu^2}{a}$	B1	N2L
	$T=\frac{9\sqrt{5}}{5}mg \quad (=4.02mg)$	M1 A1	Substitute to find T.
	Alternative method for question 4		
	$\frac{1}{2}mu^2-\frac{1}{2}m\left(\frac{u}{2}\right)^2=mg\left(a\cos\theta+a\sin\theta\right)$	*M1	Energy equation, with 2 KE terms and a two-part GPE term, allow cos/sin mix.
	$\frac{3}{4}u^2=2ag\left(\cos\theta+\sin\theta\right)$	A1	
	At B, tension in string is zero, so $mg\sin\theta=\frac{m\left(\frac{u}{2}\right)^2}{a}$ $(u^2=4ag\sin\theta)$	B1	N2L
	Eliminate θ: $u^2=\frac{8ag\sqrt{5}}{5}$	DM1 A1	
	At A, $T-mg\cos\theta=\frac{mu^2}{a}$	B1	N2L
	$T=\frac{9\sqrt{5}}{5}mg \quad (=4.02mg)$	M1 A1	Substitute to find T.
		8	

Q5

Question	Answer	Marks	Guidance
1	$T=\frac{3mgx}{a}$	B1	Their tensions equated to obtain a quadratic equation, CAO.
	$T=\frac{4mga}{3(a+x)}$	B1	
	$9x^2+9ax-4a^2=0$ leading to $(3x-a)(3x+4a)=0$	M1	
	$x=\frac{1}{3}a$	A1	
		4	

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Q6

Question	Answer	Marks	Guidance
6(a)	At A $T_A - mg\cos\theta = m\times\frac{5ag}{a}$	**B1**	N2L
	At B $T_B + mg\cos\theta = m\times\frac{v^2}{a}$	**B1**	N2L
	$\frac{1}{2}m\times 5ag - \frac{1}{2}mv^2 = mga\times 2\cos\theta$	**M1**	Energy equation with correct number of terms.
	$v^2 = 5ag - 4ga\cos\theta$	**A1**	Accept multiplied by m and/or divided by a.
	Use ratio of tensions = 9 : 5	**M1**	Use ratio and simplify to an expression in $\cos\theta$.
	$\cos\theta = \frac{2}{5}$	**A1**	CAO
		6	
6(b)	Greatest speed at lowest point $-\frac{1}{2}m\times 5ag + \frac{1}{2}mV^2 = mga\times(1-\cos\theta)$	**M1**	Energy equation including lowest point, correct number of terms.
	$V = \sqrt{\frac{31ag}{5}}$	**A1 FT**	Ft their $\cos\theta$ from part **(a)**.
		2	

Q7

Question	Answer	Marks	Guidance
3(a)	$T = 3mg$ and $T\cos\theta = mg$	**M1**	Must see both of these separately.
	Combining, $\cos\theta = \frac{1}{3}$	**A1**	At least one step of working, AG.
		2	
3(b)	$(\cos\theta = \frac{a-x}{x}$, where $x = AR)$ $AR = \frac{3}{4}a$ or $BR = \frac{1}{4}a$ or radius = $\frac{a}{\sqrt{2}}$	**B1**	$\left(\sin\theta = \frac{\sqrt{8}}{3}\right)$
	$T\sin\theta = \frac{mv^2}{r}$	**M1**	
	Combining to find an equation in v^2, a and g only.	**DM1**	
	$v^2 = 2ga$, $v = \sqrt{2ga}$	**A1**	
		4	

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Q8

Question	Answer	Marks	Guidance
7(a)	Coordinates of A: $x = a\sin 60, \quad y = a - a\cos 60$	**B1**	
	$\frac{a}{2} = \frac{a\sqrt{3}}{2}\sqrt{3} - \frac{g\frac{\left(a\sqrt{3}\right)^2}{2^2}}{2V^2 \cdot \frac{1}{4}}$	**M1**	Substitute *their* (x, y) into correct trajectory equation.
	Rearrange to find V^2.	**M1**	
	$V^2 = \frac{3}{2}ag, \quad V = \sqrt{\frac{3}{2}ag}$	**A1**	
		4	
7(b)	$\frac{1}{2}mu^2 - \frac{1}{2}mV^2 = mga(1+\cos 60)$	**M1**	Energy equation.
	$u^2 = \frac{9}{2}ag$	**A1**	u is the speed at P.
	$T - mg = \frac{m}{a}u^2$	**M1**	N2L
	$T = \frac{11}{2}mg$	**A1**	
		4	

Q9

Question	Answer	Marks
2	For A: $T = 3mg$ For B: $\uparrow T\cos\theta = mg$	**M1**
	Equate: $3mg\cos\theta = mg$ $\cos\theta = \frac{1}{3}$	**A1**
	$\rightarrow T\sin\theta = mr\omega^2$ with $r = (a-x)\sin\theta$	**M1**
	Equate: $3mg = m(a-x)\omega^2$	**A1**
	$x = \frac{a}{4}$	**A1**
		5

Q10

Question	Answer	Marks
7(a)	$(N+)mg\cos\theta = \frac{mv^2}{a}$	**B1**
	$\frac{1}{2}mv^2 - \frac{1}{2}m\frac{7ag}{2} = -mg(a + a\cos\theta)$	**M1A1**
	Loses contact when $N = 0$, so combine and simplify	**M1**
	$\cos\theta = \frac{1}{2}$: $\theta = 60°$ **AG**	**A1**
		5

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7(b)	When P is vertically below O, its horizontal displacement is $a\sin 60$, so time $T = \frac{a\sin 60}{v\cos 60} = a\sqrt{3}/v = \sqrt{\frac{6a}{g}}$	**M1**
	From **(a)**, $v^2 = \frac{1}{2}ag$	**A1**
	Vert: $h = \frac{v\sqrt{3}}{2}T - \frac{1}{2}g.T^2$	**M1**
	$\frac{3}{2}a - 3a = -\frac{3a}{2}$	**A1**
	This corresponds to the point A	**A1**
	Alternative method for question 7(b)	
	$y = x\sqrt{3} - \frac{4x^2}{a}$	**M1A1**
	Coordinates of A: $x = \frac{1}{2}a\sqrt{3},\ y = -\frac{3}{2}a$	**B1**
	Substitute coordinates into $y = x\sqrt{3} - \frac{4x^2}{a}$ and show that these satisfy this equation	**M1A1**
		5

Q11

Question	Answer	Marks
1	$T = 4mg = ma\omega^2$ so $\omega^2 = \frac{4g}{a}$	**B1**
	Time per revn = $\frac{2\pi}{\omega} = \pi\sqrt{\frac{a}{g}}$	**B1**
		2

Q12

Question	Answer	Marks
3	$T_A - mg\cos 60 = m.\frac{4ag}{a}$ $(T_A = \frac{9}{2}mg)$	**B1**
	$T_B + mg\cos 60 = m.\frac{v^2}{a}$	**B1**
	Energy: $\frac{1}{2}m(4ag - v^2) = mg.2a\cos 60$ $v^2 = 2ag$	**M1A1**
	$T_B = \frac{3mg}{2}$	**A1**
	Ratio is 3 : 1	**A1**
		6

Q13

Question	Answer	Marks	Guidance
2	At top, tension = 0, so $mg = \frac{mv^2}{a}$ $(v^2 = ag)$	**B1**	
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1+\cos\theta)$	**M1 A1**	Energy equation
	Substitute for u and v: $ag = \frac{16}{25}.5ag - 2ag(1+\cos\theta)$	**M1**	Eliminate
	$\cos\theta = \frac{1}{10}$	**A1**	
		5	

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Q14

Question	Answer	Marks	Guidance
3(a)	$T = 4mg.\dfrac{ka}{a}$	**B1**	Use Hooke's law
	$T\sin\theta = \left(\dfrac{mrg}{a}\right) = m(k+1)a\sin\theta.\dfrac{g}{a}$	**M1**	N2L horizontally. Must see T and k.
	$T = mg(k+1)$	**A1**	
	Equate: $k = \dfrac{1}{3}$	**A1**	
		4	
3(b)	$\uparrow T\cos\theta = mg$	**M1**	
	$(T = \dfrac{4}{3}mg)\quad \cos\theta = \dfrac{mg}{\frac{4}{3}mg} = \dfrac{3}{4}$	**A1**	
		2	

Q15

Question	Answer	Marks	Guidance
1	At B, $mg\cos\beta = \dfrac{mv^2}{a}$: $(v^2 = ag\cos\beta)$	**B1**	
	$\dfrac{1}{2}mv^2 = \dfrac{1}{2}mu^2 + mga(\cos\alpha - \cos\beta)$	**M1A1**	Energy equation with 4 terms and correct dimensions $(v^2 = \dfrac{3ga}{2} - 2ag\cos\beta)$
	Substitute for u, $\cos\alpha$ and v: $ag\cos\beta = \dfrac{ag}{6} + 2ag\left(\dfrac{2}{3} - \cos\beta\right)$	**M1**	Eliminate to find $\cos\beta$
	$\cos\beta = \dfrac{1}{2}$	**A1**	
		5	

Q16

Question	Answer	Marks	Guidance
4(a)	$\uparrow N\cos\theta = mg$	**B1**	
	$\leftarrow N\sin\theta = mr\sin\theta\omega^2$	**B1**	
	$\cos\theta = \dfrac{mg}{N}$ so $\cos\theta = \dfrac{g}{\omega^2 r}$	**B1**	AG
		3	
4(b)	$\cos\theta = \dfrac{r-x}{r} = \dfrac{g}{\omega^2 r}$	**B1**	Using trig of situation: must involve x
	In new situation: $r - 4x = r \times \dfrac{g}{4\omega^2 r}$	**M1**	Using new situation with $4x$ and 2ω seen
	$r - x = 4(r - 4x)$	**M1**	Combining
	$x = \dfrac{1}{5}r$	**A1**	
		4	

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Q17

Question	Answer	Marks	Guidance
11E(i)	$\frac{1}{2}mu_P^2 = \frac{1}{2}m\,(21ag/2) + mga$ $[u_P^2 = (25/2)\,ag]$	M1 A1	u_P is speed of P at lowest point, v_Q is speed of Q immediately after collision. Apply conservation of energy at lowest point (A0 if no m)
	$\frac{1}{2}\,4mv_Q^2 = 4mag$	M1	Find speed v_Q at lowest point by conservation of energy (A0 if no m)
	$v_Q = \sqrt{(2ag)}$ *or* $1{\cdot}41\sqrt{(ag)}$ *or* $4{\cdot}47\sqrt{a}$	A1	
	$mu_P = [\pm]\, mv_P + 4mv_Q$	M1	Find v_P using conservation of momentum (m may be omitted)
	$v_P = [\pm]\,(-5/\sqrt{2} + 4\sqrt{2})\,\sqrt{(ag)}$	A1	
	$\lvert v_P \rvert = (3/\sqrt{2})\,\sqrt{(ag)}$ *or* $2{\cdot}12\,\sqrt{(ag)}$ *or* $0{\cdot}671\sqrt{a}$ (AEF)	A1	Hence find speed of P
		7	
11E(ii)	V_P is speed of P when it loses contact $\frac{1}{2}mV_P^2 = \frac{1}{2}mv_P^2 - mga\,(1 + \cos\alpha)$ $[V_P^2 = (9/2)ag - 2ga\,(1 + \cos\alpha) = (5/2 - 2\cos\alpha)\,ag]$	M1 A1	Apply conservation of energy at D (A0 if no m)
	$[R_D =]\, mV_P^2/a - mg\cos\alpha = 0$ $[V_P^2 = ag\cos\alpha]$	M1 A1	Apply $F = ma$ radially at D with reaction = 0
	$(5/2 - 2\cos\alpha)\,ag = ag\cos\alpha$, $\cos\alpha = 5/6$ *or* $0{\cdot}833$	A1	Combine to find $\cos\alpha$
		5	

Q18

Question	Answer	Marks	Guidance
2(i)	$T_A = m\,v_A^2 / a - mg\cos\alpha$ $[= m\,(9ag/4) / a - mg\cos\alpha]$	B1	Find tension T_A at A from $F = ma$ radially
	$T_B = m\,v_B^2 / a + mg\cos\alpha$	B1	Find tension T_B at B from $F = ma$ radially
	$\frac{1}{2}mv_B^2 = \frac{1}{2}m\,v_A^2 + 2mga\cos\alpha$ $[v_B^2 = ag\,(9/4 + 4\cos\alpha)]$	M1 A1	Apply conservation of energy at B (A0 if no m)
	$v_B^2 = 4v_A^2 - 5ag\cos\alpha = v_A^2 + 4ga\cos\alpha$	M1	Combine using $T_B = 4\,T_A$ and $v_A^2 = 9ag/4$ to verify $\cos\alpha$
	$9ag\cos\alpha = 3v_A^2 = 27ag/4$, $\cos\alpha = 3/4$ **AG**	A1	
		6	
2(ii)	$T_B = 4\,T_A = (9 - 4\cos\alpha)\,mg = 6mg$ *or* $v_B^2 = 21ag/4$, $T_B = (21/4 + \cos\alpha)\,mg = 6mg$	M1 A1	Find T_B
		2	

Q19

Question	Answer	Marks	Guidance
4(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga\cos\theta$	M1	Use conservation of energy to slack point P_1
	$mv^2/a - mg\cos\theta = 0$	M1 A1	Equate tension at P_1 to 0 by using $F = ma$ A1 if both eqns correct, with m included. AG
	$v^2 = 2ag - 2ag\cos\theta = ag\cos\theta$	M1	Combine to verify $\cos\theta$ using $u = \sqrt{(2ag)}$
	$\cos\theta = 2/3$	A1	
		5	
4(ii)	$v_V = v\sin\theta = \sqrt{(2ag/3)}\,(\sqrt{5}/3)$ or $v_V^2 = (10/27)\,ag$	M1	Find vertical speed v_V at P_1
	$h = v_V^2/2g = (5/27)\,a$ or $0{\cdot}185\,a$	M1 A1	Find height risen above P_1 by considering vertical motion
	$h + a\cos\theta = (23/27)\,a$ or $0{\cdot}852\,a$	A1	Find total height risen above level of O
		4	

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Q20

Question	Answer	Marks	Guidance
11A(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga\cos\theta$ *and*	**M1**	Use conservation of energy (A0 if no m) *and*
	$T = mv^2/a + mg\cos\theta$ $T = mu^2/a + 3mg\cos\theta$	**M1 A1**	find tension T at P_1 by using $F = ma$ (A1 for both correct) Combine to verify $\cos\theta$
	$(11/5)mg = 2mg/5 + 3mg\cos\theta$, $\cos\theta = 3/5$ **AG**	**M1 A1**	by substituting $T = (11/5)\,mg$ and $u = \sqrt{(2ag/5)}$
	$v^2 = u^2 + 2ag\cos\theta = (2/5 + 6/5)ag = 8ag/5$ $v = \sqrt{(8ag/5)}$ *or* $\sqrt{(1{\cdot}6ag)}$ *or* $4\sqrt{a}$ (AEF)	**B1**	Find v at P_1 (can assume $\cos\theta = 3/5$)
		6	
11A(ii)	$t = (a\sin\theta)/(v\cos\theta) = 4a/3v$ [$= \frac{1}{3}\sqrt{a}$]	**M1 A1**	Find time t to P_2 by considering horizontal motion
	$h = (v\sin\theta)\,t + \frac{1}{2}gt^2$ $= 16a/15 + 5a/9 = 73a/45$ *or* $1{\cdot}62a$	**M1 A1**	Find ht. fallen at P_2 by considering vertical motion
	$OP_2 = h + a\cos\theta = 20a/9$ *or* $2{\cdot}22a$	**M1 A1**	Find OP_2
		6	

Q21

Question	Answer	Marks	Guidance
5	$\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + mga$	**M1**	Find v_1^2 at lowest point from consvn. of energy (M0 if no m)
	$v_1^2 = (12 + 2)\,ag = 14\,ag$	**A1**	
	$Mv_2 = mv_1$ with $M = (1 + k)\,m$	**M1**	Find new v_2 from consvn. of momentum
	$v_2 = v_1/(1 + k)$ [$= \sqrt{(14\,ag)}/(1 + k)$]	**A1**	
	$T_1 = mv_1^2/a + mg = (14 + 1)\,mg = 15mg$	**M1 A1**	Find tension T_1 just before collision by using $F = ma$ radially
	$T_2 = Mv_2^2/a + Mg$ $= (1 + k)\{14/(1 + k)^2 + 1\}\,mg$	**M1**	Find tension T_2 just after collision by using $F = ma$ radially (M1 needs M, not m, throughout)
	or $\{14/(1 + k) + (1 + k)\}\,mg$ (AEF)	**A1**	
	$14 + (1 + k)^2 = 15\,(1 + k)/2$	**M1**	Equate T_2 and $\frac{1}{2}\,T_1$ to give any eqn in k
	$2k^2 - 11\,k + 15 = 0$, $k = 2{\cdot}5$ *or* 3 (M1 dep)	**M1 A1**	Solve resulting quadratic for k (M1 dep on all previous M1s, and requires *quadratic* eqn)
		11	

Q22

Question	Answer	Marks	Guidance
11A(i)	$\frac{1}{2}mv_C^2 = \frac{1}{2}mu^2 + mga\,(1 - \cos\alpha)$ [$v_C^2 = u^2 + (2/5)\,ag$]	**M1A1**	Find speed v_C at C by conservation of energy (A0 if no m)
	$R_A = mg\cos\alpha + mu^2/a$	**B1**	Find reaction R_A at A by using $F = ma$
	$R_C = mg + mv_C^2/a$	**B1**	Find reaction R_C at C by using $F = ma$
	$8\{ag + u^2 + (2/5)\,ag\} = 9\{(4/5)\,ag + u^2\}$	**M1**	Verify u^2 using $R_C / R_A = 9/8$
	$u^2 = (8 + 16/5 - 36/5)\,ag = 4ag$ AG	**A1**	
		6	
11A(ii)	$\frac{1}{2}mv_B^2 = \frac{1}{2}mu^2 - 2mga\cos\alpha$ *or* $\frac{1}{2}mv_C^2 - mga\,(1 + \cos\alpha)$ [$v_C^2 = (22/5)\,ag$]	**M1**	Find speed v_B at B by conservation of energy (A0 if no m)
	$v_B^2 = (4 - 16/5)\,ag$ *or* $(22/5 - 18/5)\,ag = (4/5)\,ag$	***A**	
	$\geqslant 0$ so P reaches height of B (AEF; dep *A1)	**A1**	(or valid argument based on R_A)
	$R_A = m\,v_B^2/a - mg\cos\alpha = 4mg/5 - 4mg/5 = 0$	**M1**A1**	Find reaction R_B at B by using $F = ma$
	$\geqslant 0$ so P still [just] in contact at B (AEF; dep **A1)	**A1**	
		6	

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Q23

Question	Answer	Marks	Guidance
3(i)	$\frac{1}{2}mv_B^2 = \frac{1}{2}mu^2 - mga(\sin\alpha - \cos\alpha)$ $[v_B^2 = u^2 - (14/13)\,ag]$	**M1A1**	Find speed v_B at B by conservation of energy (A0 if no m) [$\sin\alpha = 12/13$, $\cos\alpha = 5/13$]
	$[T_B =] \; mv_B^2/a - mg\sin\alpha = 0$ $[v_B^2 = (12/13)\,ag]$	**B1**	Equate tension T_B at B to zero by using $F = ma$ radially
	$u^2 = (3\sin\alpha - 2\cos\alpha)\,ag$	**M1**	Combine to verify u^2
	$= (36/13 - 10/13)\,ag = 2\,ag$ AG	**A1**	
		5	
3(ii)	*EITHER:* $\frac{1}{2}mv_C^2 = \frac{1}{2}mu^2 + mga(1 + \cos\alpha)$ *or* $\frac{1}{2}mv_B^2 + mga(1 + \sin\alpha)$ $[v_C^2 = 62ag/13]$	**M1**	Find v_C^2 at lowest point C by conservation of energy
	$T_{max} = mv_C^2/a + mg$	**B1**	Find tension T_{max} at lowest point from $F = ma$ radially
	$= 62mg/13 + mg$	**M1**	Combine to find T_{max}
	$= 75mg/13$ *or* 5.77 *mg or* 57.7 *m*	**A1**	
	OR: $\frac{1}{2}mV^2 = \frac{1}{2}mu^2 + mga(\cos\alpha + \cos\theta)$ $[V^2 = 2ag\,(18/13 + \cos\theta)]$	**(M1**	Find V^2 at general point by conservation of energy (where OP is e.g. at θ to downward vertical)
	$T = mV^2/a + mg\cos\theta = (36/13 + 3\cos\theta)\,mg$	**B1**	Find tension T at general point from $F = ma$ radially
	$T_{max} = (36/13 + 3)\,mg$	**M1**	Combine to find T_{max} at lowest point where $\theta = 0$
	$= 75mg/13$ *or* 5.77 *mg or* 57.7 *m*	**A1)**	
		4	

Q24

Question	Answer	Marks	Guidance
5(i)	$\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + mga\cos\alpha$ $v_1^2 = ag + 2ag\cos\alpha$, $v_1 = \sqrt{(ag\,(1 + 2\cos\alpha))}$ AG	**M1 A1**	Verify v_1 for string horizontal by consvn of energy (**A0** if no m)
	Total:	**2**	
5(ii)	$T_A + mg\cos\alpha = m(\sqrt{ag})^2/a$, $T_A = mg(1 - \cos\alpha)$ $\frac{1}{2}mv_2^2 = \frac{1}{2}mu^2 + mga\cos\alpha - mg\,\frac{2}{3}\,a\cos 60°$	**M1 A1**	Find tension T_A at A from $F = ma$ radially Find v_2^2 at C by consvn. of energy (**A0** if no m)
	or $\frac{1}{2}mv_1^2 - mg\,\frac{2}{3}\,a\cos 60°$	**M1 A1**	
	$v_2^2 = ag + 2ag\cos\alpha - \frac{2}{3}ag = ag(\frac{1}{3} + 2\cos\alpha)$	**A1**	
	$T_C + mg\cos 60° = m\,v_2^2/\frac{2}{3}\,a$ [$= 3m\,v_2^2/2\,a$] $[T_C = 3mg\cos\alpha]$	**M1 A1**	Find tension T_C at C from $F = ma$ radially
	$mg(1 - \cos\alpha) = 3mg(\frac{1}{3} + 2\cos\alpha)/2 - \frac{1}{2}mg$	**M1 A1**	Find $\cos\alpha$ from $T_A = T_C$ and substituting for v_2^2
	$1 - \cos\alpha = 3\cos\alpha$, $\cos\alpha = \frac{1}{4}$	**A1**	
	Total:	**10**	

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Q25

Question	Answer	Marks	Guidance
5(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos\alpha + \sin\alpha)$ $v^2 = u^2 + 2ag(4/5 + 3/5)$ $= u^2 + 14ag/5$ AG	**M1 A1**	Verify v by conservation of energy (**A0** if no m)
	Total:	**2**	
5(ii)	$T_P = mu^2/a - mg\cos\alpha$	**B1**	Find tension T_P at P by using $F = ma$
	$T_Q = mv^2/a + mg\sin\alpha$	**B1**	Find tension T_Q at Q by using $F = ma$
	$v^2 = 2u^2 - ag(2\cos\alpha + \sin\alpha) = 2u^2 - 11ag/5$ (AEF)	**M1 A1**	Relate u^2, v^2 using $T_Q = 2T_P$ (**A0** if reqd. eqn omitted)
	$u = \sqrt{(5ag)}$ *or* $2{\cdot}24\sqrt{(ag)}$ [$v^2 = 39ag/5$]	**A1**	Eliminate v^2 to find u
	Total:	**5**	
5(iii)	$T_{min} + mg = mV^2/a$	**B1**	Find tension T_{min} at top from $F = ma$ radially
	$\frac{1}{2}mV^2 = \frac{1}{2}mu^2 - mga(1 - \cos\alpha)$ *or* $\frac{1}{2}mv^2 - mga(1 + \sin\alpha)$	**M1**	Find V^2 at top by conservation of energy (**A0** if no m)
	[$V^2 = (5 - 2/5)ag = (39/5 - 16/5)ag = 23ag/5$] $T_{min} = 23mg/5 - mg = 18mg/5$ *or* $3{\cdot}6mg$	**A1**	Combine to find T_{min}
	Total:	**3**	

Q26

Question	Answer	Marks	Guidance
11A(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(\cos\alpha + \cos\beta)$	**M1 A1**	Find v^2 at A' from conservation of energy (**A0** if no m)
	$mv^2/a = mg\cos\beta$ $u^2 = ag\cos\beta + 2ag(1/16 + \cos\beta)$	**B1**	Use $F = ma$ radially at A' with $R_{A'} = 0$ Use $\cos\alpha = 1/16$ and eliminate v^2 to verify u^2
	$= (1/8)ag(1 + 24\cos\beta)$ AG	**M1 A1**	
		5	
11A(ii)	$\frac{1}{2}mw^2 = \frac{1}{2}mu^2 - mga(1 + \cos 2\beta)$	**B1**	Find w^2 at B' from conservation of energy (**B0** if no m)
	$mw^2/a = mg\cos 2\beta$ $u^2 = ag\cos 2\beta + 2ag(1 + \cos 2\beta)$	**B1**	Use $F = ma$ radially at B' with $R_{B'} = 0$ Eliminate w^2 to find u^2
	$= ag(2 + 3\cos 2\beta)$	**M1**	
	$1 + 24\cos\beta = 8(2 + 3\cos 2\beta)$ $= 16 + 48\cos^2\beta - 24$ $16\cos^2\beta - 8\cos\beta - 3 = 0$	**M1**	Combine eqns for u^2 Formulate and solve quadratic to find $\cos\beta$
	$\cos\beta = \frac{3}{4}$ [rejecting $-\frac{1}{4}$]	**M1 A1**	
		6	
11A(iii)	$u^2 = (19/8)ag$ *or* $2{\cdot}375ag$ $R = mu^2/a + mg\cos\alpha$ $= (19/8 + 1/16)mg$	**B1**	Find u^2 using value of $\cos\beta$ Use $F = ma$ radially at A to find reaction R at A
	$= (39/16)mg$ *or* $2{\cdot}44mg$	**M1 A1**	
		3	

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Q27

Question	Mark Scheme Details		Part Mark	Total
4	Find v^2 at A from conservation of energy:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \cos\theta)$	M1 A1	
	Use $F = ma$ radially at A with $R = 0$:	$mv^2/a = mg\cos\theta$	B1	
	Use $v^2 = 3ag/5$ and eliminate $\cos\theta$ to find u:	$\cos\theta = 3/5$		
		$u^2 = v^2 + 2ag(1 + \cos\theta) = 19ag/5$		
		$u = \sqrt{(19ag/5)}$ *or* $1{\cdot}95\sqrt{(ag)}$		
	(allow numerical value of g)	*or* $\sqrt{(38a)}$ *or* $6{\cdot}16\sqrt{(a)}$	M1 A1	[5]
	Find or imply vertical component of speed at A:	$V = v\sin\theta$ [$= 4v/5$]	M1	
	Find greatest height h_A reached above A:	$h_A = V^2/2g = 24a/125$ *or* $0{\cdot}192a$	M1 A1	
	Find greatest height h_O reached above O:	$h_O = h_A + a\cos\theta$		[5]
		$= 99a/125$ *or* $0{\cdot}792a$	M1 A1	

Q28

Question Number	Mark Scheme Details		Part Mark	Total
1	Relate u^2, v^2 at lowest and highest points by energy:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - 2mga$	B1	
		[$v^2 = u^2 - 4ag$]		
	Find tension T_1 at lowest point by using $F = ma$:	$T_1 = mu^2/a + mg$	B1	
	Find tension T_2 at highest point by using $F = ma$:	$T_2 = mv^2/a - mg$	B1	
	EITHER: Relate u^2, v^2 using $T_1 = 2T_2$:	$v^2 = u^2 + 3ag$	M1	
	Eliminate v^2 to find u:	$u = \sqrt{(11ag)}$	M1 A1	[6]
	OR: Relate T_2, u^2 by substituting for v^2:	$T_2 = mu^2/a - 5mg$	(M1)	
	Use $T_1 = 2T_2$ to find u: $u = \sqrt{(11ag)}$	$v = \sqrt{(7ag)}$]	(M1 A1)	

Q29

4 (i)	Find v_1^2 at lowest point from consvn. of energy:	$\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + 2mga$		
		$\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + 2mga$	M1 A1	
	Verify new v_2 from consvn. of momentum:	$Mv_2 = mv_1$ with $M = (\lambda + 1)m$		
		$v_2 = v_1/(\lambda + 1)$		
		$= \{5/(\lambda + 1)\}\sqrt{(\frac{1}{3}ag)}$ A.G.	M1 A1	[4]
(ii)	Use $F = ma$ radially at slack point with $T = 0$, e.g.: (θ is angle of string with upward vertical)	$Mv_3^2/a = Mg\cos\theta$	B1	
	Find v_3^2 at slack point from consvn of energy:	$\frac{1}{2}Mv_3^2 = \frac{1}{2}Mv_2^2 - Mg(4a/3)$	M1 A1	
	SR: Lose max 1 mark if mass is m in either eqn.			
	Eliminate v_3^2 [$= ag/3$] using $\cos\theta = \frac{1}{3}$:	$ag/3 = v_2^2 - 8ag/3$	M1	
	Substitute for v_2 to find λ:	$3ag = \{25/(\lambda + 1)^2\}ag/3$		
		$(\lambda + 1)^2 = 25/9$, $\lambda = \frac{2}{3}$ [rejecting $-8/3$]	M1 A1	[6]
(iii)	Use $F = ma$ radially just before collision:	$T_1 = mv_1^2/a + mg$		
		[$= (25/3 + 1)mg = 28mg/3$]	B1	
	Use $F = ma$ radially just after collision:	$T_2 = Mv_2^2/a + Mg$		
		[$= (3 + 1)(5m/3)g = 20mg/3$]	B1	
	Find change in tension (either sign, AEF):	$(25mg/3)\{\lambda/(\lambda + 1)\} - \lambda mg$	A1	
		$= 8mg/3$ *or* $2{\cdot}67mg$	A1	[4]

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Q30

3	Find v^2 from conservation of energy:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1 - \cos\theta)$	M1 A1		
	Find R by using $F = ma$ radially:	$R = mg\cos\theta - mv^2/a$	B1		
	Eliminate v^2 to find R: AG	$R = mg(3\cos\theta - 2) - mu^2/a$	M1 A1	5	
	Find u^2 or v^2 in terms of $\cos\theta$ when $R = 0$:	$u^2 = ag(3\cos\theta - 2)$ *or* $v^2 = ag\cos\theta$	B1		
	EITHER: Replace $\cos\theta$ in energy eqn with $v = 2u$:	$4u^2 = u^2 + 2ag - \frac{2}{3}(u^2 + 2ag)$ *or* $u^2 + 2ag - 8u^2$	M1 A1		
	OR: Find $\cos\theta$ and substitute in energy eqn:	$[v^2/ag =]\ 4(3\cos\theta - 2) = \cos\theta$, $\cos\theta = 8/11$, $4u^2 = u^2 + 2ag(1 - 8/11)$	(M1 A1)		
	Hence find u:	$u = \sqrt{(2ag/11)}$ *or* $0.426\sqrt{(ag)}$	A	4	**9**

Q31

10A	Use conservation of energy when P is level with A:	$\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 - \frac{1}{2}mga$	B1		
	Use $F = ma$ radially at this point:	$T_1 = mv_1^2/\frac{1}{2}a$	B1		
	Eliminate v_1^2 to find T_1:	$T_1 = (2m/a)(u^2 - ag)$ **AG**	B1	3	
(i)	State or imply that string loses contact when vertical		M1		
	Use conservation of energy at this point:	$\frac{1}{2}mv_2^2 = \frac{1}{2}mu^2 - mga$	B1		
	Use $F = ma$ radially just before this point:	$T_2 = mv_2^2/\frac{1}{2}a - mg$	B1		
	Use $F = ma$ radially just after this point:	$T_3 = mv_2^2/(3a/2) - mg$	B1		
	EITHER: Find v_2^2 and hence u^2 from $T_2/T_3 = 5$:	$mv_2^2/\frac{1}{2}a - mg = 5\{mv_2^2/(3a/2) - mg\}$, $v_2^2 = 3ag$, $u^2 = 3ag + 2ag = 5ag$ **AG**	M1 A1		
	OR: Eliminate v_2^2 in $T_2/T_3 = 5$ and find u^2:	$m/a)(2u^2 - 5ag) = 5(m/a)(2u^2/3 - 7ag/3)$, $4u^2/3 = 20ag/3$, $u^2 = 5ag$ **AG**	**(**M1 A1)	6	
(ii)	Find $\cos\theta$, where $\theta = \angle AOP$, when P is level with A:	$\cos\theta = a/(3a/2) = 2/3$	B1		
	Use conservation of energy at this point:	$\frac{1}{2}mv_4^2 = \frac{1}{2}mu^2 - \frac{1}{2}mga$ *or* $\frac{1}{2}mu^2 - mg\{(3a/2)\cos\theta - \frac{1}{2}a\}$ *or* $\frac{1}{2}mv_2^2 + \frac{1}{2}mga$ *or* $\frac{1}{2}mv_2^2 + mg(3a/2)(1 - \cos\theta)$ $[v_4^2 = 4ag]$	B1		
	Use $F = ma$ radially at this point:	$T_4 = mv_4^2/(3a/2) - mg\cos\theta$	B1		
	Eliminate v_4^2 to find T_4:	$T_4 = 4mag \times 2/3a - 2mg/3 = 8mg/3 - 2mg/3 = 2mg$	M1 A1	5	**14**

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Q32

4	*EITHER:* Find tension at top from $F = ma$ vertically: $T = mu^2/a - mg$ **B1** *OR:* Use energy at e.g. θ to upward vertical: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1 - \cos\theta)$ Find tension T by using $F = ma$ radially: $T' = mv^2/a - mg\cos\theta$ Eliminate v^2: $= mu^2/a + mg(2 - 3\cos\theta)$ Find T at top by taking $\theta = 0$: $T = mu^2/a - mg$ **(B1)** Find u_{min} by requiring $T \geqslant 0$ at top [or $T > 0$]: $u^2/a - g \geqslant 0$ so $u_{min} = \sqrt{ag}$ **A.G.** **B1**		2	
	Find v at bottom from conservation of energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg \times 2a$ **M1** $v^2 = ag + 4ag$, $v = \sqrt{(5ag)}$ **A1**			
	Find new speed V from conservation of momentum: $m'V = mv$ with $m' = m + \frac{1}{4}m$ **M1** $V = 4v/5 = 4\sqrt{(ag/5)}$ *or* $(4/5)\sqrt{(5ag)}$ AEF **A1**		4	
	Find w^2 at angle θ from conservation of energy: $\frac{1}{2}m'w^2 = \frac{1}{2}m'V^2$ (condone m instead of m' here since cancels out) $- m'ga(1 + \cos\theta)$ **M1 A1** $[w^2 = ag(6/5 - 2\cos\theta)]$			
	S.R. Invalid energy method (max 2/5): $\frac{1}{2}m'w^2 = \frac{1}{2}mu^2$ [gives $T' = (5mg/4)(2 - 3\cos\theta)$] $+ mga(1 - \cos\theta)$ $- \frac{1}{4}mga(1 + \cos\theta)$ **(B1)**			
	Find tension T' there by using $F = ma$ radially: $T' = m'w^2/a - m'g\cos\theta$ **B1**			
	Eliminate w^2: $= m'V^2/a - m'g(2 + 3\cos\theta)$ **A1**			
	Substitute for m' and V: $= (5mg/4)(6/5 - 3\cos\theta)$		5	
	AEF *or* $3mg/2 - (15/4)mg\cos\theta$ **A1**			
	Find $\cos\theta$ when string becomes slack from $T' = 0$: $\cos\theta = \frac{1}{3} \times 6/5 = 2/5$ *or* 0·4 **M1 A1**		2	**13**
	S.R. Allow if found from $T' = mg(6/5 - 3\cos\theta)$			

Q33

4	**(i)**	Use conservation of energy at lowest point: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga$ B1 Use $F = ma$ radially at lowest point: $R - mg = mv^2/a$ B1 Eliminate v^2 to find R $[v^2 = 2{\cdot}3ga]$: $R = mu^2/a + 3mg = 3{\cdot}3mg$ B1	3	
	(ii)	Use conservation of energy at B to find V_B: $\frac{1}{2}mV_B^2 = \frac{1}{2}mu^2 + mga\sin\theta$ M1A1 $V_B^2 = (0{\cdot}3 + 0{\cdot}5)ga$, $V_B = \sqrt{(0{\cdot}8ga)}$ (A.E.F.) *or* $2\sqrt{(ga/5)}$ *or* $0{\cdot}894\sqrt{(ga)}$ A	3	
	(iii)	Use vertical component v_B of speed V_B at B: $v_B = V_B\cos\theta$ $[= \frac{1}{4}\sqrt{15}\, V_B = \sqrt{(\frac{3}{4}ga)}]$ Find height h reached above B: $h = v_B^2/2g = 3a/8$ M1 A1 Find height h reached above level of O: $h - a\sin\theta = 3a/8 - \frac{1}{4}a = a/8$ **A.G.** A1	4	**10**

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Q34

3	Use conservation of energy:	$\frac{1}{2}mv_B^2 = \frac{1}{2}mu^2 + 2mga\cos\alpha$ B1 $[v_B^2 = u^2 + \frac{12ag}{5}]$		
	Use $F = ma$ radially at A and B (B1 for either):	$R_A = \frac{mu^2}{a} - mg\cos\alpha$ B1 $R_B = \frac{mv_B^2}{a} + mg\cos\alpha$		
	Equate R_B to 10 R_A:	$\frac{mv_B^2}{a} + mg\cos\alpha = 10\left(\frac{mu^2}{a} - mg\cos\alpha\right)$ M1 A1		
	Eliminate v_B^2:	$u^2 + ag\cos\alpha = 10u^2 - 11ag\cos\alpha$ $[v_B^2 = \frac{17ag}{5}]$ $u^2 = (\frac{5ag}{3})\cos\alpha = ag$ **A.G.** M1 A1	6	
	Use conservation of energy for loss of contact:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos\alpha - \cos\theta)$ *or* $\frac{1}{2}mv_B^2 - mga(\cos\alpha + \cos\theta)$ B1		
	Use $F = ma$ radially with $R = 0$:	$\frac{mv^2}{a} = mg\cos\theta$ B1		
	Eliminate v^2 with $u^2 = ag$ to find $\cos\theta$:	$ga + 2ga(\cos\alpha - \cos\theta) = ga\cos\theta$ $\cos\theta = \frac{1}{3}(2\cos\alpha + 1) = \frac{11}{15}$ M1 A1	4	**[10]**

Q35

Question	Answer	Marks	Guidance
4	Tcos60 = 0.3g	**M1**	Resolve vertically
	T = 6 N	**A1**	
	T = 16e/0.8 (= 6) leads to e = 0.3	**M1**	Use T =λx/L
	r = (0.8 + 0.3)sin60 (= 1.1sin60)	**A1**	
	Tsin60 = 0.3 v^2 /(1.1sin60)	**M1**	Use N2L horizontally
	v = 4.06 m s^{-1}	**A1**	
		6	

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Q36

Question	Answer	Marks	Guidance
1(i)	$T\cos\theta\left(=T\times\frac{0.15}{0.8}\right)=0.3g$	**M1**	Resolve vertically. θ is the angle between the string and the vertical
	$T=16\,N$ **AG**	**A1**	
		2	
1(ii)	$r^2=0.8^2-0.15^2$	**B1**	$r=0.78581...$
	$16\sin\theta\left(=16\times\frac{0.78581...}{0.8}\right)=\frac{0.3v^2}{0.78581...}$	**M1**	Use Newton's Second Law horizontally
	$v=6.416$	**A1**	
		3	

Q37

Question	Answer	Marks	Guidance
3(i)	$r=0.4$ m	**B1**	Use Pythagoras's theorem
	$T\cos\theta=0.4\times5^2\times0.4$	**M1**	Use Newton's Second Law
	$T\times\frac{0.4}{0.5}=4, T=5N$	**A1**	
		3	
3(ii)	$R=0.4g-T\sin\theta$	**M1**	Resolve vertically. Allow for their T for M1
	$R=1N$	**A1**	
		2	

Q38

Question	Answer	Marks	Guidance
6(i)	$0.3^2+r^2=0.5^2$ hence $r=0.4$	**B1**	Use Pythagoras's theorem
	$8\times0.4=3.2\ \text{m s}^{-1}$	**B1**	Use $v=r\omega$
		2	
6(ii)	$A\times\frac{3}{5}-B\times\frac{3}{5}=0.3g$	**B1**	Resolve vertically
	$A\times\frac{4}{5}+B\times\frac{4}{5}=0.3\times8^2\times0.4$ or $\frac{0.3\times3.2^2}{0.4}$	**M1A1**	Use Newton's Second Law horizontally
		M1	Attempt to solve for B
	$B=2.3$ N	**A1**	
	$2.3=\frac{46(0.5-L)}{L}$	**M1**	Use $T=\frac{\lambda x}{l}$ and attempt to solve
	$L=0.476$ m or $\frac{10}{21}$	**A1**	
		7	

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Q39

Question	Answer	Marks	Guidance
5(i)	$r = 0.5\sin 30$ (= 0.25 m)	**B1**	
	$T\cos 30 - T\cos 70 = 0.4g$	**M1**	Resolve vertically
	$T = 7.6335..$	**A1**	
	$7.6335\sin 30 + 7.6335\sin 70 = 0.4v^2 / 0.25$	**M1**	Use Newton's Second Law with $a = \frac{v^2}{r}$
	$v = 2.62$ m s^{-1}	**A1**	
		5	
5(ii)	$A\cos 30 - B\cos 70 = 0.4g$ **and** $A\sin 30 + B\sin 70 = 0.4 \times 12^2 \times 0.5\sin 30$	**M1**	Resolves vertically and uses Newton's Second Law with $a = r\omega^2$
		A1	Both correct
		M1	Attempt to solve for A or B
	$A = 8.82$ N	**A1**	
	$B = 10.6$ N	**A1**	
		5	

Q40

Question	Answer	Marks	Guidance
5(i)	*EITHER:* $R\cos 60 = 0.3g$	**(M1**	Resolve vertically
	$R = 6$ N	**A1**	
	$6\cos 30 = 0.3\omega^2 \times 0.4\cos 30$	**M1**	Use Newton's Second Law horizontally
	$\omega = 5\sqrt{2} = 7.07$ rad s^{-1}	**A1)**	
	OR: $0.3g\cos 30 = 0.3 \times (0.4\cos 30)\omega^2\cos 60$	**(M1**	Resolve along the tangent
		A1	Correct equation
	$\omega = 5\sqrt{2} = 7.07$ rad s^{-1}	**M1**	Attempt to solve for ω
		A1)	
		4	
5(ii)	$R\cos 60 = 0.3g + 5\sin 30$	**M1**	Resolve vertically
	$R = 11$ N	**A1**	
	$11\cos 30 + 5\cos 30 = 0.3v^2 / (0.4\cos 30)$	**M1**	Resolve horizontally
	$v = 4$ m s^{-1}	**A1**	
		4	

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Q41

Question	Answer	Marks	Guidance
6(i)	r [= 0.6 – (0.4-0.3)]= 0.5	**B1**	
	$T = 0.3g$	**B1**	Resolve vertically for Q
	$0.2v^2 / 0.5 = 0.3g$	**M1**	Use Newton's Second Law horizontally for P
	$v = 2.74$ ms^{-1}	**A1**	
		4	
6(ii)	$r = 0.5 + e$	**B1**	e = extension of the string
	$T = \frac{15e}{0.3} = 50e$	**B1**	Use $T = \lambda x/l$
	$0.2 \times 8^2 (5 + e) = 50e + 0.3g$	**M1**	Use Newton's Second Law horizontally with $a = r\omega^2$
	$e = \frac{(6.4 - 3)}{(50 - 12.8)}$ (= 0.0914)	**A1**	
	HP = 0.591 m	**A1**	
		5	

Q42

6(i)	cosθ = 0.5 and sinθ = $\sqrt{3}$ /2	**B1**	θ is the angle that AP makes with the horizontal. Note tanθ = $\sqrt{3}$
	Tsinθ = 0.2 g	**M1**	Resolve vertically for P. Note tension in BP is zero
	Tcosθ = 0.2 ω^2 × 0.3	**M1**	Use Newton's Second Law horizontally
	ω = 4.39 rad s^{-1}	**A1**	
	Total:	**4**	
6(ii)	T_A sinθ = 0.2 g + T_B sinθ	**M1**	Resolve vertically for P
	T_A sinθ = 0.2 g + 5sinθ	**M1**	Use T_B = 5
	T_A = 7.309	**A1**	
	5cosθ + 7.309cosθ = 0.2 v^2 /0.3	**M1**	Use Newton's Second Law horizontally
	v = 3.04 m s^{-1}	**A1**	
	Total:	**5**	

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Q43

Question	Answer	Marks	Guidance
5(i)	$0.1\times1.5^2/0.4 = T\cos\theta$	**M1**	Note r = 0.4, $\cos\theta = 0.8$, $\sin\theta = 0.6$ Use Newton's Second Law horizontally
	T = 0.703	**A1**	
	$R = 0.1g - T\sin\theta$	**M1**	Resolve vertically for P
	R = 0.578	**A1**	
		4	
5(ii)	$T + T\sin\theta = 0.1g$	**M1**	Resolve vertically for P
	T = 0.625	**A1**	
	$0.1\omega^2\times0.4 = 0.625\cos\theta$	**M1**	Use Newton's Second Law horizontally
	$\omega = 3.54$ rad s^{-1}	**A1**	
		4	

Q44

Question	Answer	Marks	Guidance
7(i)	$T\cos30 + U\cos30 = 0.1g$ (= 1)	**B1**	Resolve vertically for P Note T and U are the tensions in PR and PB respectively
	$T\cos60 - U\cos60 = 0.1 \times 6^2 \times 0.3$ (= 1.08)	**M1A1**	Use Newton's Second Law horizontally
	$2T\cos30\cos60 = 1.08\cos30 + 1\cos60$	**M1**	Attempt to eliminate U
	T (= 1.65735) = 1.66 N **AG**	**A1**	
		5	
7(ii)	$F - T\cos60 = 0.2 \times 6^2 \times 0.15$ **or** $R = 0.2g + T\cos30$	**M1**	Use Newton's Second Law horizontally or resolve vertically
	F (= 1.9086) = 1.91 N	**A1**	
	R (= 3.4353) = 3.44 N	**A1**	
	$\mu = 1.9086/3.4353$	**M1**	Use $F = \mu R$
	$\mu = 0.556$	**A1**	Accept $\mu = 0.56$
		5	

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Q45

Question	Answer	Marks	Guidance
5(i)	$T\cos60 = 1.5 + 0.4g$	**M1**	Resolve vertically for *P*
	$T = 11$ N	**A1**	
	$T\sin60 = 0.4\omega^2 \text{x} 0.5\sin60$	**M1**	Uses Newton's Second Law horizontally for *P*
	$\omega = \sqrt{55} = 7.42$	**A1**	
	Total:	**4**	
5(ii)	$m = 0.15$ (from $mg = 1.5$)	**B1**	Resolves vertically for Q
	$T^* = 0.15 \text{ x } 7.42^2 \text{ x } 0.5\sin60$	**M1**	Uses Newton's Second Law horizontally for Q
	$T^* = 3.57$ N	**A1**	
	Total:	**3**	

Q46

Question	Answer	Marks	Guidance
6(i)	$T = 12 \times 0.1 / 0.4$ (= 3 N)	**B1**	Uses $T = \lambda x / L$
	$3\sin\theta = 0.15\,\omega^2 (0.5\sin\theta)$	**M1**	Uses Newton's Second Law horizontally
	$\omega = 6.32$ rad s^{-1}	**A1**	
	$T\cos\theta = 0.15g$ $(\cos\theta = 0.5)$	**M1**	Resolves vertically
	$\theta = 6$	**A1**	
	Total:	**5**	
6(ii)	$v = 6.32 \times 0.5\sin 60$	**B1 FT**	Uses $v = r\omega$ and r = 0.5sin60
	KE = $0.15(6.32 \times 0.5\sin 60)^2 / 2$ (=0.5625J)	**B1**	
	Difference = $0.5625 - 12 \times 0.1^2 / (2 \times 0.4)$	**M1**	Uses EE = $\lambda x^2 / (2L)$
	Difference = 0.4125 J	**A1**	
	Total:	**4**	

Q47

2(i)	$6\cos 60 = 4\cos 60 + mg$	**M1**	Resolve vertically
	$m = 0.1$ kg	**A1**	
	Total:	**2**	
2(ii)	radius = 0.7sin60	**B1**	
	$6\sin 60 + 4\sin 60 = 0.1\,v^2 / (0.7\sin 60)$	**M1**	Uses Newton's Second Law horizontally with 3 terms
	$v = 7.25$ m s^{-1}	**A1**	
	Total:	**3**	

Q48

Question	Answer	Marks	Guidance
1	$R = 0.4 \times 6^2 \times 0.5$ (= 7.2 N)	**B1**	Uses Newton's Second Law horizontally and $a = r\omega^2$.
	$F = 0.4\,g$	**B1**	Resolve vertically.
	$\mu = 4/7.2$	**M1**	Use $F = \mu R$.
	$\mu = 0.556$ or 5/9	**A1**	Accept $\mu = 0.56$.
		4	

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Q49

Question	Answer	Marks	Guidance
3(i)	$T\sin60 + R = 0.6g$	**M1**	Resolves vertically.
	$T\cos60 = 0.6 \times 0.5^2/(0.4\cos60)$	**M1**	Uses Newton's Second Law horizontally.
	$T = 1.5$	**A1**	
	$R = 4.7(0)$ N	**A1**	
		4	
3(ii)	$T\sin60 = 0.6g$ (leads to $T = 6.9282...$)	**M1**	Resolve vertically. Note $R = 0$.
	$6.9282...\cos60 = 0.6\,v^2/(0.4\cos60)$	**M1**	Use Newton's second Law horizontally.
	$v = 1.07$	**A1**	Greatest value.
		3	

Q50

Question	Answer	Marks	Guidance
6(i)	$T = 0.2 \times 2.5^2/(0.4 + e)$	**B1**	Uses Newton's Second Law towards the centre of the circle.
	$T = 8e/0.4$	**B1**	Uses $T = \lambda x/L$.
	$1.25/(0.4 + e) = 20e \rightarrow 20\,e^2 + 8e - 1.25 = 0$	**M1**	Eliminates T to find e.
	$e = 0.12(0)$ m	**A1**	
		4	
6(ii)	$0.2\,v^2/2 = 2[8\,x^2/(2 \times 0.4)]$	**B1**	Uses $KE = 2EE$.
	$0.2\,v^2/(0.4 + x) = 8x/0.4$	**B1**	Uses $T = \lambda x/L$ and $T = m\,v^2/r$.
		M1	Attempts to solve the 2 equations to find v or x.
	$x = 0.4$ and $v = 5.66$ or $4\sqrt{2}$	**A1A1**	
		5	

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Q51

7 (i)	$R\cos60 + T\cos30 = 0.2\,g$	**M1**		Resolving vertically
	$R + T\sqrt{3} = 4$ AG	**A1**	2	g = 10 must be used
(ii)	$T\sin30 - R\sin60 = 0.2\,\omega^2 \times 0.6\sin60$	**M1**	2	N2L horizontally with accn= ω^2 r
	$(T - R\sqrt{3} = 0.12\,\omega^2\sqrt{3})$	**A1**		Accept with trig ratios
(iii) (a)	$R\cos60\sin30 + R\sin60\cos30 = 2\sin30 - 0.2 \times 2^2 \times 0.6\sin60\cos30$	**M1**		Substitutes ω=2 and eliminates T from **(i)** and **(ii)**
	$R = 0.64\,N$	**A1**	2	Accept answers between 0.639 and 0.641 inclusive
	OR			
	$R + 3R = 4 - 0.12 \times 2^2 \times 3$	**M1**		
	$R = 0.64$	**A1**		
(b)	$T\cos30 = 2$	**M1**		
	$T = 2.31$	**A1**		When R = 0, T = $4\sqrt{3}$ /3 or $4/\sqrt{3}$
	$2.31\sin30 = 0.2\,\omega^2 \times 0.6\sin60$ AND $v = \omega \times 0.6\sin60$	**M1**		
	$v = 1.73\ m\,s^{-1}$	**A1**	4	
	OR			
	$2.31\sin30 = 0.2\,v^2/(0.6\sin60)$	**M1**		
	$v = 1.73\ m\,s^{-1}$	**A1**		Final pair of marks

Q52

6 (i) (a)	$2\cos45 + 2 \times 3/5 = 0.4\,\omega^2 \times 0.3$	**M1** **A1**		Uses N2L with 2 components of T and accn = $0.3\,\omega^2$
	$\omega = 4.67\ rad\,s^{-1}$	**A1**	3	
(i) (b)	$R + 2\sin45 + 2 \times 4/5 = 0.4\,g$	**M1**		
	$R = 0.986\,N$	**A1**	2	
(ii)	$T\sin45 + T(4/5) = 0.4\,g$	**M1**		
	$T = 2.65$	**A1**		2.654
	$T\cos45 + T(3/5) = 0.4\,v^2/0.3$	**M1**		
	$v = 1.61\ m\,s^{-1}$	**A1**	4	

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Q53

6 (i)	Bcos30 − Asin30 = 0.2g	**B1**		Resolving vertically for P
	Bcos60 − Acos30 = $0.2\times1.2^2/0.4$	**M1**		2 components of tension, N2L with accn = v^2/r
		A1		
		M1		Attempts to eliminate one unknown
	A = 0.753 N	**A1**		
	B = 2.74 N	**A1**	6	
(ii)	r = 0.8sin60	**B1**		
		M1		Resolves vertically or uses N2L horizontally
	Bcos60 − Acos30 = 0.2g	**A1**		
	Bcos30 − Acos60 = $0.2\times5^2\times0.8\sin60$	**A1**		
		M1		For solving to find A
	A = 0	**A1**	6	

Q54

1	T = 12 N	**B1**		T = 12(2L–L)/L
	T = 0.3 x 4^2/r	**M1**		Accn = v^2/r
	12 = 4.8/(2L)	**A1**√		ft candidates expression for T
	L = 0.2	**A1**	4	

Q55

5 (i)	$\theta(=tan^{-1}0.45/0.6 = 36.87..) = 36.9°$	**B1**		Or $\tan\theta = 3/4$
	$0.4v^2/0.6 = 5\cos\theta$	**M1**		
	$v = 2.45\,\text{m}s^{-1}$	**A1**	3	Or $\sqrt{6}$
(ii)	$T\sin\theta = 0.4g$	**M1**		
	$T = 6.67\,\text{N}$	**A1**		Accept 0.66, $6\frac{2}{3}$, 20/3
	$0.4\omega^2$ x 0.6 = $6.67\cos\theta$	**M1**		
	$\omega = 4.71\ \text{rad}\,s^{-1}$	**A1**	4	Accept $4.72\ \text{rad}\,s^{-1}$

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Q56

7	**(i)**	$R\cos30 = 0.5g$		**M1**	
		$R = 5.77(35...)$ AG		**A1**	
		$R\sin30 = 0.5\omega^2$ x 0.4		**M1**	
		$\omega = 3.8(0)$ rad s^{-1}		**A1**	[4]
	(ii)	$T = 36(0.5 – 0.45) /0.45$		**M1**	4 N
		Vert cmpt = 4 x 0.3/0.5 = 2.4		**A1**	
		Horiz cmpt = 4 x 0.4/0.5 = 3.2		**A1**	
		$R\cos30 +2.4 = 0.5g$		**M1**	
		$R = 3(.00…)$ N			
		$0.5v^2 /0.4 =3.2 + R\sin30$		**M1**	
		$v = 1.94$ m s^{-1}		**A1**	[6]

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Hooke's Law

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Q1

3 One end of a light elastic string, of natural length a and modulus of elasticity kmg, is attached to a fixed point A. The other end of the string is attached to a particle P of mass $4m$. The particle P hangs in equilibrium a distance x vertically below A.

(a) Show that $k = \dfrac{4a}{x-a}$. [1]

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An additional particle, of mass $2m$, is now attached to P and the combined particle is released from rest at the original equilibrium position of P. When the combined particle has descended a distance $\frac{1}{3}a$, its speed is $\frac{1}{3}\sqrt{ga}$.

(b) Find x in terms of a. [6]

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Q2

2 One end of a light elastic string of natural length 0.8 m and modulus of elasticity 36 N is attached to a fixed point O on a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of O. The particle is projected with speed $\sqrt{2}\,\text{m s}^{-1}$ directly down the plane from the position where OP is equal to the natural length of the string.

Find the maximum extension of the string during the subsequent motion. [5]

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Q3

3 A light elastic string has natural length a and modulus of elasticity $12mg$. One end of the string is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle hangs in equilibrium vertically below O. The particle is pulled vertically down and released from rest with the extension of the string equal to e, where $e > \frac{1}{3}a$. In the subsequent motion the particle has speed $\sqrt{2ga}$ when it has ascended a distance $\frac{1}{3}a$.

Find e in terms of a. [6]

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Q4

2 A light spring AB has natural length a and modulus of elasticity $5mg$. The end A of the spring is attached to a fixed point on a smooth horizontal surface. A particle P of mass m is attached to the end B of the spring. The spring and particle P are at rest on the surface.

Another particle Q of mass km is moving with speed $\sqrt{4ga}$ along the horizontal surface towards P in the direction BA. The particles P and Q collide directly and coalesce. In the subsequent motion the greatest amount by which the spring is compressed is $\frac{1}{5}a$.

Find the value of k. [6]

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Q5

3 One end of a light elastic spring, of natural length a and modulus of elasticity $5mg$, is attached to a fixed point A. The other end of the spring is attached to a particle P of mass m. The spring hangs with P vertically below A. The particle P is released from rest in the position where the extension of the spring is $\frac{1}{2}a$.

(a) Show that the initial acceleration of P is $\frac{3}{2}g$ upwards. [3]

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(b) Find the speed of P when the spring first returns to its natural length. [4]

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Q6

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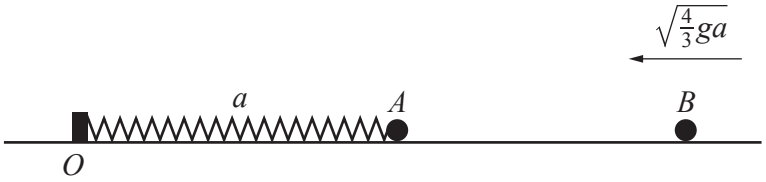

One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O. The other end of the spring is attached to a particle A of mass km, where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a. A second particle B, of mass m, is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).

The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

(a) Find the value of k. [6]

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At the point C the horizontal surface becomes rough, with coefficient of friction μ between the combined particle and the surface. The deceleration of the combined particle at C is $\frac{9}{20}g$.

(b) Find the value of μ. [4]

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Q7

1 A particle P of mass m is placed on a fixed smooth plane which is inclined at an angle θ to the horizontal. A light spring, of natural length a and modulus of elasticity $3mg$, has one end attached to P and the other end attached to a fixed point O at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.

Find, in terms of a and θ, an expression for the greatest extension of the spring in the subsequent motion. [3]

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Q8

6 One end of a light elastic string, of natural length a and modulus of elasticity k, is attached to a particle P of mass m. The other end of the string is attached to a fixed point Q. The particle P is projected vertically upwards from Q. When P is moving upwards and at a distance $\frac{4}{3}a$ directly above Q, it has a speed $\sqrt{2ga}$. At this point, its acceleration is $\frac{7}{3}g$ downwards.

Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by P. [8]

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Q9

5 A particle P of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point O. The particle P is released from rest at the point 0.4 m vertically below O.

(i) Find the greatest speed of P. [5]

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(ii) Calculate the greatest distance of P below O. [3]

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5 A particle P of mass 0.4 kg is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 6 N. The other end of the string is attached to a fixed point O. The particle P is released from rest at the point $(0.5 + x)$ m vertically below O. The particle P comes to instantaneous rest at O.

(i) Find x. [3]

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(ii) Find the greatest speed of P. [5]

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Q11

5 A light elastic string has natural length a m and modulus of elasticity λ N. When the length of the string is 1.6 m the tension is 4 N. When the length of the string is 2 m the tension is 6 N.

(i) Find the values of a and λ. [5]

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One end of the string is attached to a fixed point O on a smooth horizontal surface. The other end of the string is attached to a particle P of mass 0.2 kg. The particle P moves with constant speed on the surface in a circle with centre O and radius 1.9 m.

(ii) Find the speed of P. [3]

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Q12

5 A particle P of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 9 N. The other end of the string is attached to a fixed point O on a smooth plane inclined at 30° to the horizontal. OA is a line of greatest slope of the plane with A below the level of O and OA = 0.8 m. The particle P is released from rest at A.

(i) Find the initial acceleration of P. [4]

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(ii) Find the greatest speed of P. [5]

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Q13

1 A particle of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 9 N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. The particle is projected horizontally from O with speed $4\ \text{m s}^{-1}$. Find the greatest distance of the particle from O. [3]

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Q14

3 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point O. The particle P is projected vertically downwards with speed 2 m s^{-1} from the point 0.5 m vertically below O. For an instant when the acceleration of P is 4 m s^{-2} downwards, find the extension of the string and the speed of P. [6]

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Q15

3 A small ball B is connected to one end of a light elastic string of natural length 0.4 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point A. The ball is projected with speed 1 m s^{-1} vertically downwards from a position 0.4 m vertically below A, and reaches its greatest speed at the point 0.7 m below A.

(i) Show that the mass of B is 0.9 kg. [2]

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(ii) Calculate the greatest speed of B. [4]

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Q16

5 A particle P of mass 0.7 kg is attached by a light elastic string to a fixed point O on a smooth plane inclined at an angle of 30° to the horizontal. The natural length of the string is 0.5 m and the modulus of elasticity is 20 N. The particle P is projected up the line of greatest slope through O from a point A below the level of O. The initial kinetic energy of P is 1.8 J and the initial elastic potential energy in the string is also 1.8 J.

(i) Find the distance OA. [2]

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(ii) Find the greatest speed of P in the motion. [6]

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2 One end of a light elastic string is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.4 kg. The string has natural length 0.6 m and modulus of elasticity 24 N. The particle is released from rest at O. Find the two possible values of the distance OP for which the particle has speed $1.5\ \text{m s}^{-1}$. [6]

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Q18

3 A particle P of mass 0.4 kg is attached to a fixed point O by a light elastic string of natural length 0.5 m and modulus of elasticity 20 N. The particle P is released from rest at O.

(i) Find the greatest speed of P in the subsequent motion. [4]

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(ii) Find the distance below O of the point at which P comes to instantaneous rest. [3]

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Q19

5 A particle P of mass 0.7 kg is attached to a fixed point O by a light elastic string of natural length 0.6 m and modulus of elasticity 15 N. The particle P is projected vertically downwards from the point A, 0.8 m vertically below O. The initial speed of P is $2\,\text{m s}^{-1}$.

(i) Find the distance below A of the point at which P comes to instantaneous rest. [4]

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(ii) Find the greatest speed of P in the motion. [4]

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Q20

7 One end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.4 kg which hangs in equilibrium vertically below O.

(i) Calculate the extension of the string. [2]

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P is projected vertically downwards from the equilibrium position with speed 5 m s^{-1}.

(ii) Calculate the distance P travels before it is first at instantaneous rest. [4]

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When P is first at instantaneous rest a stationary particle of mass 0.4 kg becomes attached to P.

(iii) Find the greatest speed of the combined particle in the subsequent motion. [4]

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Q21

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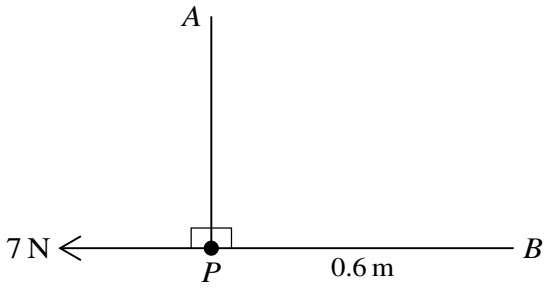

Fig. 1

One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a particle P of mass m kg which hangs vertically below A. The particle is also attached to one end of a light elastic string of natural length 0.25 m. The other end of this string is attached to a point B which is 0.6 m from P and on the same horizontal level as P. Equilibrium is maintained by a horizontal force of magnitude 7 N applied to P (see Fig. 1).

(i) Calculate the modulus of elasticity of the elastic string. [2]

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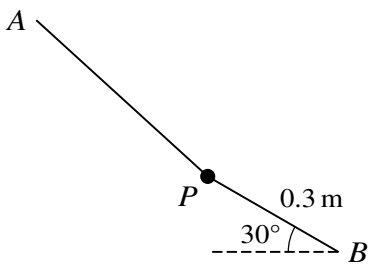

Fig. 2

P is released from rest by removing the 7 N force. In its subsequent motion P first comes to instantaneous rest at a point where $BP = 0.3$ m and the elastic string makes an angle of 30° with the horizontal (see Fig. 2).

(ii) Find the value of m. [4]

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Q22

1 A particle P of mass 0.2 kg moves with speed 4 m s^{-1} and angular speed 5 rad s^{-1} in a horizontal circle on a smooth surface. P is attached to one end of a light elastic string of natural length 0.6 m. The other end of the string is attached to the point on the surface which is the centre of the circular motion of P.

(i) Find the radius of this circle. [1]

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(ii) Find the modulus of elasticity of the string. [4]

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Q23

5 A particle of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 6 N. The other end of the string is attached to a fixed point O. The particle is projected vertically downwards from O with initial speed $2\,\text{m}\,\text{s}^{-1}$.

(i) Calculate the greatest speed of the particle during its descent. [5]

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(ii) Find the greatest distance of the particle below O. [3]

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Q24

5 One end of a light elastic string of natural length 0.8 m and modulus of elasticity 24 N is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.3 kg. P is projected vertically upwards with speed 4 m s^{-1} from a position 1.2 m vertically below O.

(i) Calculate the speed of the particle at the position where it is moving with zero acceleration. [5]

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(ii) Show that the particle moves 1.2 m while moving upwards with constant deceleration. [3]

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Q25

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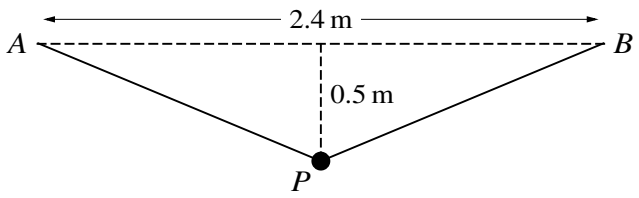

A light elastic string has natural length 2 m and modulus of elasticity 39 N. The ends of the string are attached to fixed points A and B which are at the same horizontal level and 2.4 m apart. A particle P of mass m kg is attached to the mid-point of the string and hangs in equilibrium at a point 0.5 m below AB (see diagram).

(i) Show that $m = 0.9$. [4]

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P is projected vertically downwards from the equilibrium position, and comes to instantaneous rest at a point 1.6 m below AB.

(ii) Calculate the speed of projection of P. [5]

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5 A particle P of mass 0.6 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point A, and P hangs in equilibrium.

(i) Calculate the extension of the string. [2]

P is projected vertically downwards from the equilibrium position with speed $4.5\,\text{m s}^{-1}$.

(ii) Find the distance AP when the speed of P is $3.5\,\text{m s}^{-1}$ and P is below the equilibrium position. [4]

(iii) Calculate the speed of P when it is 0.5 m above the equilibrium position. [3]

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Q27

7 A particle P is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point O on a smooth plane inclined at an angle of 30° to the horizontal. P rests in equilibrium on the plane, 1.6 m from O.

(i) Calculate the mass of P. [2]

A particle Q, with mass equal to the mass of P, is projected up the plane along a line of greatest slope. When Q strikes P the two particles coalesce. The combined particle remains attached to the string and moves up the plane, coming to instantaneous rest after moving 0.2 m.

(ii) Show that the initial kinetic energy of the combined particle is 1 J. [4]

The combined particle subsequently moves down the plane.

(iii) Calculate the greatest speed of the combined particle in the subsequent motion. [5]

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[Turn over

2 One end of a light elastic string of natural length 0.4 m is attached to a fixed point O. The other end of the string is attached to a particle of weight 5 N which hangs in equilibrium 0.6 m vertically below O.

(i) Find the modulus of elasticity of the string. [2]

The particle is projected vertically upwards from the equilibrium position and comes to instantaneous rest after travelling 0.3 m upwards.

(ii) Calculate the speed of projection of the particle. [3]

(iii) Calculate the greatest extension of the string in the subsequent motion. [3]

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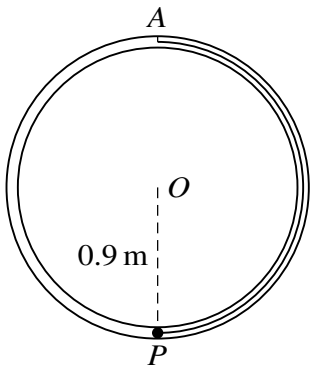
Q29

6



The diagram shows a smooth narrow tube formed into a fixed vertical circle with centre O and radius 0.9 m. A light elastic string with modulus of elasticity 8 N and natural length 1.2 m has one end attached to the highest point A on the inside of the tube. The other end of the string is attached to a particle P of mass 0.2 kg. The particle is released from rest at the lowest point on the inside of the tube. By considering energy, calculate

(i) the speed of P when it is at the same horizontal level as O, [4]

(ii) the speed of P at the instant when the string becomes slack. [3]

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[Turn over

2 A particle P of mass 0.5 kg is attached to one end of a light elastic string with modulus of elasticity 24 N and natural length 0.6 m. The other end of the string is attached to a fixed point A. The particle P hangs in equilibrium vertically below A.

(i) Find the distance AP. [2]

The particle P is raised to A and released from rest.

(ii) Calculate the greatest speed of P in the subsequent motion. [3]

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Hooke's Law MS

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Q1

Question	Answer	Marks	Guidance
3(a)	Use Hooke's Law: $4mg = \frac{kmg(x-a)}{a}$ leading to $k = \frac{4a}{x-a}$	**B1**	AG. Shown convincingly.
		1	
3(b)	Gain in KE + gain in EPE = loss in GPE	**B1**	One correct EPE term seen.
	$\frac{1}{2}\times 6m\times\frac{ga}{9}+\frac{1}{2}\frac{kmg}{a}\left(\left(x+\frac{a}{3}-a\right)^2-(x-a)^2\right)=6mg\times\frac{a}{3}$	**M1** **A1**	All 3 types of energy included in energy equation. All terms correct.
	Simplify and substitute for k from part **(a)**	**M1**	
	Obtain linear equation in x and a	**M1**	
	$x=\frac{5}{3}a$	**A1**	($k = 6$)
		6	

Q2

Question	Answer	Marks	Guidance
2	$\frac{1}{2}\times\frac{36x^2}{0.8}$ $(=22.5x^2)$	**B1**	EPE correct.
	Loss in GPE + loss in KE = gain in EPE $x\,mg\sin\alpha+\frac{1}{2}m\times 2=\frac{1}{2}\times\frac{36x^2}{0.8}$	***M1**	Energy equation with only GPE, EPE and KE terms, allow sign errors, allow missing g for M1 only, weight must be resolved (allow sin or cos).
	All terms correct	**A1**	
	$\left(\frac{6}{5}x+\frac{1}{5}=\frac{9}{4}x^2\right)$ [leading to $45x^2-24x-4=0$]	**DM1**	Simplify to 3-term quadratic and attempt to solve.
	$(3x-2)(15x+2)=0$ $x=\frac{2}{3}$ only	**A1**	
		5	

Q3

Question	Answer	Marks	Guidance
3	Loss in EPE = $\frac{1}{2}\times\frac{12mge^2}{a}-\frac{1}{2}\times\frac{12mg}{a}\times\left(e-\frac{a}{3}\right)^2$ $\left(=\frac{2mg}{3}(6e-a)\right)$	**B1**	Either term correct.
	Gain in KE = $\frac{1}{2}mv^2$ and Gain in GPE = $\frac{mga}{3}$	**B1**	
	Gain in KE + Gain in GPE = Loss in EPE	**M1**	KE, GPE and at least one EPE term.
	$\frac{1}{2}mv^2+\frac{mga}{3}=\frac{2mg}{3}(6e-a)$	**A1**	All terms correct.
	Simplify to a linear equation in e.	**M1**	
	$e=\frac{1}{2}a$	**A1**	
		6	

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Q4

Question	Answer	Marks	Guidance
2	At the collision of P and Q: $(m+km)v=kmu$	**M1**	Momentum conserved, allow missing k on RHS.
	So $v=\frac{k\sqrt{4ga}}{(1+k)}$	**A1**	
	EPE = $\frac{1}{2}\times\frac{5mg}{a}\times\left(\frac{a}{5}\right)^2 \quad \left(=\frac{mga}{10}\right)$	**B1**	
	Loss in KE = Gain in EPE: $\frac{1}{2}m(k+1)v^2=\frac{1}{2}\times\frac{5mg}{a}\times\left(\frac{a}{5}\right)^2$	**M1**	Energy equation, LHS correct, EPE dimensionally correct.
	Substitute for v and rearrange to form quadratic equation in k $20k^2=1+k$	**M1**	
	$k=\frac{1}{4}$	**A1**	
		6	

Q5

Question	Answer	Marks
3(a)	$T-mg=m.a$	**M1**
	$T=5mg.\frac{1}{2}a/a\ =\frac{5}{2}mg$	**M1**
	$a=\frac{3}{2}g$ (upwards) **AG**	**A1**
		3
3(b)	Gain in KE = $\frac{1}{2}mv^2$ Gain in GPE = $\frac{1}{2}mga$	**B1**
	Loss in EPE = $\frac{1}{2}\frac{5mg.\left(\frac{1}{2}a\right)^2}{a}$	**B1**
	$\frac{1}{2}mv^2+\frac{1}{2}mga=\frac{1}{2}\frac{5mg.\left(\frac{1}{2}a\right)^2}{a}$ $[\Rightarrow \frac{1}{2}mv^2+\frac{1}{2}mga=\frac{5}{8}mga\ ]$	**M1**
	$v=\frac{1}{2}\sqrt{ga}$	**A1**
		4

Q6

Question	Answer	Marks
7(a)	Collision: $(k+1)mv=mu$ so $v=\frac{u}{k+1}$	**B1**
	Loss in KE = $\frac{1}{2}m(k+1)\left[v^2-\frac{1}{4}v^2\right]$	**B1**
	Gain in EPE = ½ .4mg/a. $\left(\frac{a}{4}\right)^2$	**B1**
	So, $\frac{1}{2}.\frac{3}{4}.m(k+1)\frac{u^2}{(k+1)^2}=\frac{mga}{8}$	**M1**
	Use value of u and solve	**M1**
	$k=3$	**A1**
		6

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7(b)	$T + F = 4m.\frac{9g}{20}$	**M1**
	$4mg.\frac{\frac{a}{4}}{a} + F = \frac{9mg}{5}$	**M1**
	$\mu R = 4mg / 5$ and compare with $R = 4mg$	**B1**
	$\mu = \frac{1}{5}$	**A1**
		4

Q7

Question	Answer	Marks	Guidance
1	Gain in EPE = $\frac{1}{2}.\frac{3mgx^2}{a}$	**B1**	EPE gain.
	Loss in GPE = $mgx\sin\theta$ Equate	**M1**	Equate energies
	$x = \frac{2}{3}a\sin\theta$	**A1**	Using forces scores B0M0A0
		3	

Q8

Question	Answer	Marks	Guidance
6	$T + mg = m.\frac{7}{3}g$	**M1**	
	With $T = k.\frac{\frac{a}{3}}{a}$ giving $k = 4mg$	**A1**	AG
	Let greatest height above Q be $\frac{4}{3}a + x$ Gain in GPE = mgx and Loss in KE = ½ $m.2ga$	**B1**	The length being found may be expressed as the total extension of the string or the greatest height above Q. GPE and KE
	Gain in EPE = $\frac{1}{2}.\frac{4mg}{a}\left(\left(x+\frac{a}{3}\right)^2 - \left(\frac{a}{3}\right)^2\right)$	**B1**	EPE Note: initial EPE = $\frac{2mga}{9}$
	$\frac{4mg}{2a}\left(x^2 + \frac{2ax}{3} + \frac{a^2}{9} - \frac{a^2}{9}\right) + mgx = mga$	**M1 A1**	Energy equation, correct number of terms
	$2x^2 + \frac{7ax}{3} - a^2 = 0$	**M1**	Simplify to quadratic
	$x = \frac{1}{3}a$ so greatest height is $\frac{5}{3}a$	**A1**	

Q9

Question	Answer	Marks	Guidance
5(i)	0.3g = 24e/0.6	**M1**	Note greatest speed occurs at the equilibrium position. Use T=λx/L
	e = 0.075 m	**A1**	Fall = 0.275 m
	PE Change = 0.3g × 0.275	**B1**	
	$0.3v^2/2 = 0.3g \times 0.275 - 24 \times 0.075^2/(2 \times 0.6)$	**M1**	Set up a 3 term energy equation
	v = 2.18 m s^{-1}	**A1**	
		5	
5(ii)	$0.3g(0.2 + E) = 24E^2/(2 \times 0.6)$	**M1**	Set up an energy equation. Note v = 0 at the greatest distance
	$20E^2 - 3E - 0.6 = 0$	**M1**	Attempt to solve a 3 term quadratic equation
	E = 0.264 and so greatest distance is 0.864 m	**A1**	
		3	

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Q10

Question	Answer	Marks	Guidance
5(i)	$0.4g(0.5+x)=\frac{6x^2}{(2\times0.5)}$	**M1**	Set up an energy equation
	$6x^2-4x-2=0$ or $3x^2-2x-1=0$	**M1**	Attempt to solve a 3 term quadratic equation
	$x=1$ (ignore $-\frac{1}{3}$ if seen)	**A1**	
		3	
5(ii)	$0.4g=\frac{6e}{0.5}$	**M1**	Use $T=\frac{\lambda x}{l}$ to find the extension at the equilibrium position
	$e=\frac{1}{3}$	**A1**	
	PE change $=0.4g\left(0.5+\frac{1}{3}\right)$	**B1ft**	Ft for candidate's e
	$\frac{0.4V^2}{2}=0.4g\left(0.5+\frac{1}{3}\right)-\frac{6\left(\frac{1}{3}\right)^2}{(2\times0.5)}$	**M1**	Set up a three term energy equation
	$V=3.65$ ms^{-1}	**A1**	
		5	

Q11

Question	Answer	Marks	Guidance
5(i)	$4=\frac{\lambda(1.6-a)}{a}$	**B1**	Use $T=\left(\frac{\lambda x}{l}\right)$ twice
	$6=\frac{\lambda(2-a)}{a}$	**B1**	
	$1.5=\frac{(2-a)}{(1.6-a)}$	**M1**	Attempt to solve the simultaneous equations
	$0.4=0.5(a)$, $a=0.8$	**A1**	
	$\lambda=4$	**A1**	
		5	
5(ii)	$T=4\times\frac{1.1}{0.8}(=5.5)$	**B1**	**FT** Use $T=\frac{\lambda x}{L}$, ft candidates λ and a
	$5.5=\frac{0.2v^2}{1.9}$	**M1**	Use Newton's Second Law horizontally
	$v=7.23$ ms^{-1}	**A1**	
		3	

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Q12

Question	Answer	Marks	Guidance
5(i)	$T = \frac{9\times(0.8-0.6)}{0.6}$	**M1**	Use $T = \frac{\lambda x}{l}$. Note $OP = \frac{0.4}{\sin 30}$
	$T = 3$ N	**A1**	
	$0.3a = 3 - 0.3g\sin 30$	**M1**	Use Newton's Second Law along the slope
	$a = 5\ \text{m}\,s^{-1}$	**A1**	
		4	
5(ii)	$0.3g\sin 30 = \frac{9e}{0.6}$	**M1**	Note the maximum speed is at the equilibrium position
	$e = 0.1$	**A1**	
	$\text{EPE} = \frac{9\times(0.8-0.6)^2}{2\times 0.6}$ or $\frac{9\times 0.1^2}{2\times 0.6}$	**B1**	
	$\frac{0.3v^2}{2} = \frac{9\times(0.8-0.6)^2}{2\times 0.6} - \frac{9\times 0.1^2}{2\times 0.6} - 0.3g\times 0.1\sin 30$	**M1**	Set up a 4 term energy equation
	$v = 0.707\ \text{m s}^{-1}$	**A1**	
		5	

Q13

Question	Answer	Marks	Guidance
1	$\frac{0.3\times 4^2}{2} = \frac{9e^2}{2\times 0.6}$	**M1**	Set up an energy equation. Note the final velocity is zero.
	$e = 0.566$ or $\frac{2\sqrt{2}}{5}$	**A1**	
	Distance = 1.17 m	**A1**	
		3	

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Q14

Question	Answer	Marks	Guidance
3	$0.5\times 4 = 0.5g - T$	**M1**	Use Newton's Second Law vertically
	$T = \frac{12e}{0.6}$	**M1**	Use $T = \frac{\lambda x}{l}$
	$e\left(=\frac{3\times 0.6}{12}\right) = 0.15$	**A1**	
	$\text{EPE} = \frac{12\times 0.15^2}{2\times 0.6}$ and distance fallen $= 0.6 - 0.5 + 0.15$	**B1ft**	
	$\frac{0.5v^2}{2} = \frac{0.5\times 2^2}{2} + 0.5g(0.6-0.5+0.15) - \frac{12\times 0.15^2}{2\times 0.6}$	**M1**	Set up a 4 term energy equation
	$v = 2.85\ \text{m s}^{-1}$	**A1**	
		6	

Q15

Question	Answer	Marks	Guidance
3(i)	$mg = 12(0.7 - 0.4) / 0.4$	**M1**	Use $T = \lambda x / L$
	$m = 0.9$ kg **AG**	**A1**	
		2	

3(ii)	EPE = $12(0.7 - 0.4)^2 / (2 \times 0.4)$	**B1**	Correct EPE term
	$0.9v^2 / 2 = 0.9g(0.7 - 0.4) + 0.9 \times 1^2 / 2 - 12(0.7 - 0.4)^2 / (2 \times 0.4)$	**M1**	Attempts a 4 term energy equation
		A1	Correct equation
	$v = 2$ m s^{-1}	**A1**	
		4	

Q16

Question	Answer	Marks	Guidance
5(i)	$1.8 = \dfrac{20e^2}{(2 \times 0.5)}$	**M1**	Use $T = \dfrac{\lambda x}{l}$
	$e = 0.3$, OA = 0.8	**A1**	
		2	
5(ii)	$0.7g\sin 30 = \dfrac{20x}{0.5}$	**M1**	Use Newton's Second Law up the plane
	$x = 0.0875$ m	**A1**	
	$\text{EPE} = \dfrac{20 \times 0.0875^2}{(2 \times 0.5)}$	**B1**	
	$\dfrac{0.7v^2}{2} = 1.8 + 1.8 - 0.7g(0.3 - 0.0875)\sin 30 - \dfrac{20 \times 0.0875^2}{(2 \times 0.5)}$	**M1**	Attempt to set up a 5 term energy equation
		A1	Correct equation
	$v = 2.78$ ms^{-1}	**A1**	
		6	

Q17

Question	Answer	Marks	Guidance
2	EPE = $24(x - 0.6)$ $/(2 \times 0.6)$	**B1**	Correct EPE term. Note x = OP
	$0.4 \times 1.5^2 / 2 = 0.4gx - 24(x - 0.6)^2 / (2 \times 0.6)$ [$20x^2 - 28x + 7.65 = 0$ or equivalent]	**M1**	Attempt to find a 3 term energy equation
		M1	Attempt to solve the 3 term quadratic equation
	OP = 1.0279 m, 0.372 m (reject)	**A1**	Correct answer chosen
	$0.4 \times 1.5^2 / 2 = 0.4gx$	**M1**	Note the particle is moving upwards and the string is slack
	OP = 0.1125 m	**A1**	
	Total:	**6**	

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	Alternative method		
	EPE = $24x^2/(2 \times 0.6)$	**B1**	x is the extension
	$0.4 \times 1.5^2/2 = 0.4g(x + 0.6) - 24x^2/(2 \times 0.6)$ $[20x^2 - 4x - 1.95 = 0$ or equivalent]	**M1**	Attempt to find a 3 term energy equation
		M1	Attempt to solve the 3 term quadratic equation
	[$x = 0.42787, -0.22787$ .reject] OP = 0.6 + 0.42787 = 1.0279	**A1**	
	$0.4 \times 1.5^2/2 = 0.4g(x + 0.6)$ [$x = -0.4875$]	**M1**	Note the particle is moving upwards and the string is slack
	OP = 0.6 – 0.4875 = 0.1125	**A1**	
	Total:	**6**	

Q18

Question	Answer	Marks	Guidance
3(i)	$20e/0.5 = 0.4g$	**M1**	Use $T = \lambda x/L$
	$e = 0.1$	**A1**	
	$0.4v^2/2 = 0.4g(0.5 + 0.1) - 20 \times 0.1^2/(2 \times 0.5)$	**M1**	Attempt to set up a 3 term energy equation
	$v = \sqrt{11} = 3.32$	**A1**	
		4	
3(ii)	0 + x) = $20x^2/(2 \times 0.5)$	**M1**	Attempt to set up a 2 term energy equation
	$[0 = 20x^2 - 4x - 2]$ [$x = 0.432$]	**M1**	Attempt to solve a 3 term quadratic equation
	Distance below O = (0.5 + 0.432) = 0.932 m	**A1**	
		3	

Q19

Question	Answer	Marks	Guidance
5(i)	15 + $x)^2/(2 \times 0.6)$ **or** $15 \times 0.2^2/(2 \times 0.6)$	**B1**	
		M1	Set up a 4 term energy equation
	$15(0.2 + x)^2/(2 \times 0.6) = 15 \times 0.2^2/(2 \times 0.6) + 0.7gx + (0.7 \times 2^2)/2$	**A1**	$0.5 + 5x + 12.5x^2 = 0.5 + 7x + 1.4$
	$x = 0.424$ m	**A1**	
		4	
5(ii)	$0.7g = 15e/0.6$	**M1**	Use $T = \lambda x/L$
	$e = 0.28$ m	**A1**	
	$(0.7 \times 2^2)/2 + 0.7g(0.28 - 0.2) + (15 \times 0.2^2)/(2 \times 0.6)$ $= (0.7 \times v^2)/2 + (15 \times 0.28^2)/(2 \times 0.6)$	**M1**	Set up a 5 term energy equation
	$v = 2.06$ m s^{-1}	**A1**	
		4	

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Q20

Question	Answer	Marks	Guidance
7(i)	$g = 24e/0.6$	**M1**	Uses $T = \lambda x/L$
	$e = 0.1$ m	**A1**	
	Total:	**2**	
7(ii)	Initial EE = $24 \times 0.1^2/(2 \times 0.6)$ (= 0.2 J)	**B1**	Uses EE = $\lambda x^2/2L$
	$0.4 \times 5^2/2 + 0.4gd = 24(0.1 + d)^2/(2 \times 0.6) - 24 \times 0.1^2/(2 \times 0.6)$	**M1 A1**	Set up a 4 term energy equation involving EE, PE and KE
	$d = 0.5$ m	**A1**	
	Total:	**4**	
7(iii)	$e = 0.2$	**B1**	
	$0.8v^2/2 = 24 \times 0.6^2/(2 \times 0.6) - 24 \times 0.2^2/(2 \times 0.6) - 0.8g \times 0.4$	**M1 A1**	Set up a 4 term energy equation in EE, PE and KE
	$v = 2\sqrt{2} = 2.83$ ms^{-1}	**A1**	
	Total:	**4**	

Q21

2(i)	$7 = 0.35\lambda / 0.25$	**M1**	Uses $T = \lambda x / L$
	$\lambda =$	**A1**	
	Total:	**2**	
2(ii)	EE = $0.35^2 \times 5 / (2 \times 0.25)$ or $0.05^2 \times 5 / (2 \times 0.05)$	**B1**	Uses EE = $\lambda x^2 / 2L$
	PE = $mg \times 0.3\sin 30$	**B1**	
	$mg \times 0.3\sin 30 = 0.35^2 \times 5 / (2 \times 0.25) - 0.05^2 \times 5 / (2 \times 0.25)$	**M1**	Sets up a 3 term energy equation involving EE, KE and PE
	$m = 0.8$	**A1**	
	Total:	**4**	

Q22

Question	Answer	Marks	Notes
1(i)	$(4 = 5r)$ $r = 0.8$ m	**B1**	Uses $v = r\omega$
	Total:	**1**	
1(ii)	$T = 0.2 \times 5^2 \times 0.8$	**M1**	Uses Newton's Second Law horizontally
	$T = 4$ N	**A1 FT**	FT with their radius from part (i)
	$4 = \lambda(0.8 - 0.6) / 0.6$	**M1**	Uses $T = \lambda x / L$
	$\lambda = 1$	**A1**	
	Total:	**4**	

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Q23

Question	Answer	Marks	Notes
5(i)	$0.3g = 6e / 0.8$	**M1**	Uses $T = \lambda x / L$
	$e = 0.4$ m	**A1**	
	EE $= 6 \times 0.4^2/(2 \times 0.8)$	**B1 FT**	FT for their e
	$0.3v^2/2 - 0.3 \times 2^2/2 = 0.3\,g(0.8 + 0.4) - 6 \times 0.4^2/(2 \times 0.8)$	**M1**	Sets up a 4 term energy equation involving EE, KE and PE
	$v = 4.9(0)$ m s^{-1} or $2\sqrt{6}$	**A1**	
	Total:	**5**	
5(ii)	$0.3 \times 2^2/2 + 0.3gL = 6(L - 0.8)^2/(2 \times 0.8)$	**M1**	Sets up a 3 term energy equation involving EE, KE and PE
		A1	
	$L = 2.18$ m	**A1**	Ignore answers less than 0.8
	Total:	**3**	

Q24

Question	Answer	Marks	Guidance
5(i)	$g = 24e$	**M1**	Use $T = \lambda x/L$
	$e = 0.1$	**A1**	
	$EE = 24 \times (1.2 - 0.8)^2/(2 \times 0.8)$ or $24 \times 0.1^2/(2 \times 0.8)$	**B1**	Use EE $= \lambda x^2/(2L)$.
	$0.3v^2/2 = 0.3 \times 4^2/2 + 24 \times (1.2 - 0.8)^2/(2 \times 0.8)$ $-24 \times 0.1^2/(2 \times 0.8) - 0.3g(1.2 - 0.8)$	**M1**	Sets up a 5 term energy equation involving *EE*, *KE* and *PE*.
	$v = 5$ m s^{-1}	**A1**	
		5	
5(ii)	$0.5 \times 5^2/2 + 24 \times 0.1^2/(2 \times 0.8) = 0.3(x + 0.9) \times 10$	**M1**	Sets up a 3 term energy equation where x is the distance above 0 when $v = 0$.
	$x = 0.4$	**A1**	
	Distance moved $= 0.8 + 0.4 = 1.2$ m	**A1**	AG
		3	

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Q25

Question	Answer	Marks	Guidance
4(i)	$e = \sqrt{(0.5^2 + 1.2^2)} - 1 = 0.3$	**B1**	
	$T = 39 \times 0.3/1$	**M1**	Uses $T = \lambda x/L$.
	$mg = 2 \times (39 \times 0.3/1) \times 0.5/1.3$	**M1**	Resolves vertically.
	$m = 0.9$	**A1**	AG
		4	
4(ii)	$E = \sqrt{(1.6^2 + 1.2^2)} - 1 = 1$ m	**B1**	E = extension when the particle comes to instantaneous rest.
	$EE = 39 \times 1^2/(2 \times 1)$ or $39 \times 0.3^2/(2 \times 1)$	**B1**	
	$0.9v^2/2 + 0.9g(1.6 - 0.5)$ $= 2[39 \times 1^2/(2 \times 1) - 39 \times 0.3^2/(2 \times 1)]$	**M1A1**	Set up a 4 term energy equation involving EE, KE and PE.
	$v = 7.54$ m s^{-1}	**A1**	
		5	

Q26

5 (i)	$24e/0.8 = 0.2g$ $e = 0.2$	**M1** **A1**	 2	
(ii)	$24 \times 0.2^2 / (2 \times 0.8)$ (= 0.6)	**B1**√		ft(cv0.2) Initial EE
	$0.6 \times 4.5^2/2 + 0.6gd + 24 \times 0.2^2/(2 \times 0.8)$ $= 0.6 \times 3.5^2/2 + 24 \times (0.2 + d)^2/(2 \times 0.8)$	**M1** **A1**		PE/EE/KE balance attempt d = distance particle falls
	$d = 0.4$ so AP ($= 0.8 + 0.2 + 0.4$) = 1.4m	**A1**	4	
(iii)	$24 \times 0.2^2/(2 \times 0.8) + 0.6 \times 4.5^2/2 =$ $0.6v^2/2 + 0.6g \times 0.5$	**M1** **A1**		PE/EE/KE balance, 4 terms. Award B1ft for initial KE if not already seen in part ii
	$v = 3.5$ m s^{-1}	**A1**	3	

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Q27

7 (i)	$12(1.6–1.2)/1.2 = mg\sin30$	**M1**		Uses $T = \lambda ext/l$
	$m = 0.8\,kg$	**A1**	2	
(ii)	PE change = 1.6	**B1**√		2 × ans(i)
		B1		Both EE terms correct
	IKE + $12 \times 0.4^2/2.4$ = $1.6 \times 0.2g\sin30 + 12 \times 0.2^2/2.4$	**M1**		KE/PE/EE balance Both EE terms correct
	IKE = 1 J **AG**	**A1**	4	
(iii)	$12e/1.2 = 1.6\,g\sin30$	**M1**		$\lambda e \times t/l$ = new weight component
	$e = 0.8$	**A1**		May be stated without explanation
	$1.6v^2/2 + 12 \times 0.8^2/2.4$ = $1.6g \times 0.6\sin30 + 12 \times 0.2^2/2.4$	**M1** **A1**		Must use new equilibrium position
	$v = 1.5\ m\,s^{-1}$	**A1**	5	

Q28

2 (i)	$5 = 0.2\lambda/0.4$	**M1**		Tension = $\lambda ext/l$
	$\lambda = 10\,N$	**A1**	2	
(ii)		**B1**√		Correct EE term
	$10(0.2^2/(2 \times 0.4) + (5/g)v^2)/2 = 0.3 \times 5$	**M1**		PE/KE/EE 3 terms
	$v = 2\ m\,s^{-1}$	**A1**	3	
(iii)		**B1**√		Correct EE term
	$10e^2/(2 \times 0.4) = 5(e + 0.1)$	**M1**		Energy equation
	$e = 0.483$	**A1**	3	

Q29

6 (i)	EE = $8(0.9\pi – 1.2)^2/(2 \text{ x } 1.2)$	**B1**		Initial EE = 8.83 J
	$8.83 = 0.2g \text{ x } 0.9 + 0.2v^2/2 + 8(0.9\pi/2 – 1.2)^2/(2 \text{ x } 1.2)$	**M1** **A1**		
	$v = 8.29\ m\,s^{-1}$	**A1**	4	
(ii)	$\theta = 1.2/0.9 = 4/3$ rad (=76.4°)	**B1**		
	$8.83 = 0.2g \text{ x } 0.9 + 0.2g \text{ x } 0.9\cos\theta + 0.\ v^2/2$	**M1**		$0.2 \text{ x } 8.29^2/2 = 0.2g \text{ x } 0.9\cos\theta + 0.\ v^2/2$
	$v = 8.13\ m\,s^{-1}$	**A1**	3	

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Q30

2 (i)	$5 = 24e / 0.6$ AP = 0.725 m	**M1** **A1**	[2]	Hence $e = 0.125$
(ii)	$24 \times 0.125^2/2 \times 0.6$ $0.5g \times 0.725 =$ $24 \times 0.125^2/ 2 \times 0.6 + 0.5v^2 /2$ $v = 3.64\,\text{m s}^{-1}$	**B1** **M1** **A1**	[3]	EE at eqm (= 0.3125) KE/EE/PE conservation

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Linear Motion Under Variable Force

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Q1

1 A particle P of mass 1 kg is moving along a straight line against a resistive force of magnitude $\frac{10\sqrt{v}}{(t+1)^2}$ N, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of P at time t s. When $t = 0$, $v = 25$.

Find an expression for v in terms of t. [5]

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Q2

5 A particle P of mass m kg is projected vertically upwards from a point O, with speed $20\,\text{m s}^{-1}$, and moves under gravity. There is a resistive force of magnitude $2mv$ N, where $v\,\text{m s}^{-1}$ is the speed of P at time t s after projection.

(a) Find an expression for v in terms of t, while P is moving upwards. [6]

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The displacement of P from O is x m at time t s.

(b) Find an expression for x in terms of t, while P is moving upwards. [2]

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(c) Find, correct to 3 significant figures, the greatest height above O reached by P. [2]

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Q3

2 A particle P of mass m kg moves along a horizontal straight line with acceleration $a\,\text{m s}^{-2}$ given by

$$a = \frac{v(1-2t^2)}{t},$$

where $v\,\text{m s}^{-1}$ is the velocity of P at time t s.

(a) Find an expression for v in terms of t and an arbitrary constant. [3]

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(b) Given that $a = 5$ when $t = 1$, find an expression, in terms of m and t, for the horizontal force acting on P at time t. [3]

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Q4

6 A particle P of mass 2 kg moves along a horizontal straight line. The point O is a fixed point on this line. At time t s the velocity of P is $v\,\text{m s}^{-1}$ and the displacement of P from O is x m.

A force of magnitude $\left(8x - \frac{128}{x^3}\right)$ N acts on P in the direction OP. When $t = 0$, $x = 8$ and $v = -15$.

(a) Show that $v = -\frac{2}{x}(x^2 - 4)$. [5]

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(b) Find an expression for x in terms of t. [4]

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Q5

5 A particle P is moving along a straight line with acceleration $3ku - kv$ where v is its velocity at time t, u is its initial velocity and k is a constant. The velocity and acceleration of P are both in the direction of increasing displacement from the initial position.

(a) Find the time taken for P to achieve a velocity of $2u$. [3]

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(b) Find an expression for the displacement of P from its initial position when its velocity is $2u$. [5]

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2 A particle Q of mass m kg falls from rest under gravity. The motion of Q is resisted by a force of magnitude mkv N, where $v\,\text{ms}^{-1}$ is the speed of Q at time t s and k is a positive constant.

Find an expression for v in terms of g, k and t. [6]

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Q7

7 A particle P moving in a straight line has displacement x m from a fixed point O on the line at time t s. The acceleration of P, in m s^{-2}, is given by $\frac{200}{x^2} - \frac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and P has velocity $10\,\text{m s}^{-1}$ directed towards O.

(a) Show that the velocity $v\,\text{m s}^{-1}$ of P is given by $v = \frac{10(1-2x)}{x}$. [5]

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(b) Show that x and t are related by the equation $e^{-40t} = (2x-1)e^{2x-2}$ and deduce what happens to x as t becomes large. [5]

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Q8

7 A particle P of mass m kg moves in a horizontal straight line against a resistive force of magnitude mkv^2 N, where $v\,\text{m s}^{-1}$ is the speed of P after it has moved a distance x m and k is a positive constant. The initial speed of P is $u\,\text{m s}^{-1}$.

(a) Show that $x = \frac{1}{k}\ln 2$ when $v = \frac{1}{2}u$. [4]

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Beginning at the instant when the speed of P is $\frac{1}{2}u$, an additional force acts on P. This force has magnitude $\frac{5m}{v}$N and acts in the direction of increasing x.

(b) Show that when the speed of P has increased again to $u\,\mathrm{m\,s^{-1}}$, the total distance travelled by P is given by an expression of the form

$$\frac{1}{3k}\ln\left(\frac{A-ku^3}{B-ku^3}\right),$$

stating the values of the constants A and B. [7]

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Q9

7 A particle P is projected horizontally from a point O on a rough horizontal surface. The coefficient of friction between the particle and the surface is 0.2. A horizontal force of magnitude $0.06t$ N directed away from O acts on P, where t s is the time after projection. P comes to rest when $t = 4$.

(i) The particle begins to move again when $t = 8$. Show that the mass of P is 0.24 kg. [2]

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(ii) Show that, for $0 \leqslant t \leqslant 4$, $\frac{\mathrm{d}v}{\mathrm{d}t} = 0.25t - 2$, and find the speed of projection of P. [5]

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(iii) Find the distance from O at which P comes to rest. [4]

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Q10

7 A particle P of mass 0.5 kg is attached to a fixed point O by a light elastic string of natural length 1 m and modulus of elasticity 16 N. The particle P is projected vertically upwards from O with speed $6\,\text{m s}^{-1}$. A resisting force of magnitude $0.1x^2$ N acts on P when P has displacement x m above O. After projection the upwards velocity of P is $v\,\text{m s}^{-1}$.

(i) Show that, before the string becomes taut, $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -10 - 0.2x^2$. [2]

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(ii) Find the velocity of P at the instant the string becomes taut. [4]

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(iii) Find an expression for the acceleration of P while it is moving upwards after the string becomes taut. [2]

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(iv) Verify that P comes to instantaneous rest before the extension of the string is 0.5 m. [4]

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Q11

4 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 16 N. The other end of the string is attached to a fixed point O. The particle P is released from rest at the point 0.8 m vertically below O. When the extension of the string is x m, the downwards velocity of P is v m s^{-1} and a force of magnitude $25x^2$ N opposes the motion of P.

(i) Show that, when P is moving downwards, $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 10 - 40x - 50x^2$. [2]

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(ii) For the instant when P has its greatest downwards speed, find the kinetic energy of P and the elastic potential energy stored in the string. [6]

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Q12

3 A smooth horizontal surface has two fixed points O and A which are 0.8 m apart. A particle P of mass 0.25 kg is projected with velocity 3 m s^{-1} horizontally from A in the direction away from O. The velocity of P is v m s^{-1} when the displacement of P from O is x m. A force of magnitude kv^2x^{-2} N opposes the motion of P.

(i) Show that $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -4kv^2x^{-2}$. [1]

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(ii) Express v in terms of k and x. [5]

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Q13

6 A particle P of mass 0.2 kg is projected horizontally from a fixed point O on a smooth horizontal surface. When the displacement of P from O is x m the velocity of P is $v\,\text{m s}^{-1}$. A horizontal force of variable magnitude $0.09\sqrt{x}$ N directed away from O acts on P. An additional force of constant magnitude 0.3 N directed towards O acts on P.

(i) Show that $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 0.45\sqrt{x} - 1.5$. [2]

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(ii) Find the value of x for which the acceleration of P is zero. [2]

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(iii) Given that the minimum value of v is positive, find the set of possible values for the speed of projection. [5]

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Q14

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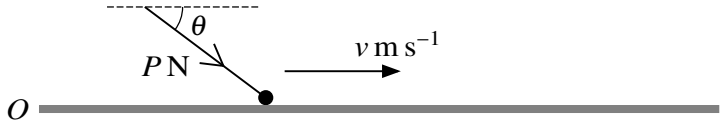

A small object of mass 0.2 kg rests at a point O on a rough horizontal surface. The coefficient of friction between the object and the surface is 0.5. A force of magnitude P N acting at an angle θ below the horizontal is applied to the object. The velocity of the object is v m s^{-1} away from O at time t s after the force begins to act (see diagram). It is given that $\tan\theta = \frac{3}{4}$ and that $P = 0.4t$ for $0 \leqslant t \leqslant 8$.

(i) Find the value of t when the object starts to move. [3]

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(ii) Show that, when the force is acting and the object is in motion, $\frac{\mathrm{d}v}{\mathrm{d}t} = t - 5$. [2]

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When $t = 8$ the force of magnitude P N ceases to act.

(iii) Find the distance travelled by the object after $t = 8$ before it comes to rest. [5]

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Q15

3 A particle P of mass 0.4 kg is projected horizontally along a smooth horizontal plane from a point O. At time t s after projection the velocity of P is v m s^{-1}. A force of magnitude $0.8t$ N directed away from O acts on P and a force of magnitude $2e^{-t}$ N opposes the motion of P.

(i) Show that $\frac{dv}{dt} = 2t - 5e^{-t}$. [2]

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(ii) Given that $v = 8$ when $t = 1$, express v in terms of t. [3]

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(iii) Find the speed of projection of P. [2]

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Q16

7 A particle P of mass 0.2 kg is released from rest at a point O above horizontal ground. At time t s after its release the velocity of P is v m s^{-1} downwards. A vertically downwards force of magnitude $0.6t$ N acts on P. A vertically upwards force of magnitude $k\mathrm{e}^{-t}$ N, where k is a constant, also acts on P.

(i) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 10 - 5k\mathrm{e}^{-t} + 3t$. [2]

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(ii) Find the greatest value of k for which P does not initially move upwards. [3]

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(iii) Given that $k = 1$, and that P strikes the ground when $t = 2$, find the height of O above the ground. [5]

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Q17

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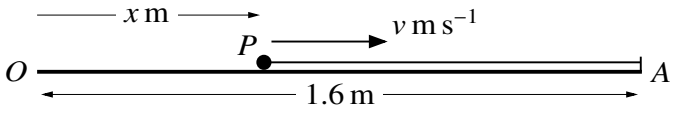

A particle P of mass 0.5 kg is projected along a smooth horizontal surface towards a fixed point A. Initially P is at a point O on the surface, and after projection, P has a displacement from O of x m and velocity v m s^{-1}. The particle P is connected to A by a light elastic string of natural length 0.8 m and modulus of elasticity 16 N. The distance OA is 1.6 m (see diagram). The motion of P is resisted by a force of magnitude $24x^2$ N.

(i) Show that $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 32 - 40x - 48x^2$ while P is in motion and the string is stretched. [3]

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The maximum value of v is 4.5.

(ii) Find the initial value of v. [5]

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Q18

3 A particle P of mass 0.4 kg is projected horizontally along a smooth horizontal plane from a point O. After projection the velocity of P is $v\,\text{m s}^{-1}$ and its displacement from O is x m. A force of magnitude $8x$ N directed away from O acts on P and a force of magnitude $(2e^{-x} + 4)$ N opposes the motion of P. One end of a light elastic string of natural length 0.5 m is attached to O and the other end of the string is attached to P.

(i) Show that $v\dfrac{dv}{dx} = 20x - 10 - 5e^{-x}$ before the elastic string becomes taut. [2]

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(ii) Given that the initial velocity of P is $6\,\text{m s}^{-1}$, find v when the string first becomes taut. [3]

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When the string is taut, the acceleration of P is proportional to e^{-x}.

(iii) Find the modulus of elasticity of the string. [2]

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Q19

6 O and A are fixed points on a rough horizontal surface, with $OA = 1$ m. A particle P of mass 0.4 kg is projected horizontally with speed $U\,\text{m s}^{-1}$ from A in the direction OA and moves in a straight line. After projection, when the displacement of P from O is x m, the velocity of P is $v\,\text{m s}^{-1}$. The coefficient of friction between the surface and P is 0.4. A force of magnitude $\frac{0.8}{x}$ N acts on P in the direction PO.

(i) Show that, while the particle is in motion, $v\frac{\mathrm{d}v}{\mathrm{d}x} = -4 - \frac{2}{x}$. [3]

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It is given that P comes to instantaneous rest between $x = 2.0$ and $x = 2.1$.

(ii) Find the set of possible values of U. [5]

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Q20

7 A particle P of mass 0.5 kg is at rest at a point O on a rough horizontal surface. At time $t = 0$, where t is in seconds, a horizontal force acting in a fixed direction is applied to P. At time t s the magnitude of the force is $0.6t^2$ N and the velocity of P away from O is v m s^{-1}. It is given that P remains at rest at O until $t = 0.5$.

(i) Calculate the coefficient of friction between P and the surface, and show that

$$\frac{dv}{dt} = 1.2t^2 - 0.3 \quad \text{for } t > 0.5. \qquad [3]$$

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(ii) Express v in terms of t for $t > 0.5$. [3]

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(iii) Find the displacement of P from O when $t = 1.2$. [3]

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Q21

4 A small object of mass 0.4 kg is released from rest at a point 8 m above the ground. The object descends vertically and when its downwards displacement from its initial position is x m the object has velocity v m s^{-1}. While the object is moving, a force of magnitude $0.2v^2$ N opposes the motion.

(i) Show that $v\frac{\mathrm{d}v}{\mathrm{d}x} = 10 - 0.5v^2$. [2]

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(ii) Express v in terms of x. [4]

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(iii) Find the increase in the value of v during the final 4 m of the descent of the object. [2]

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Q22

7 A particle P of mass 0.2 kg is released from rest at a point O on a rough plane inclined at 60° to the horizontal, and travels down a line of greatest slope. The coefficient of friction between P and the plane is 0.3. A force of magnitude $0.6x$ N acts on P in the direction PO, where x m is the displacement of P from O.

(i) Show that $v\frac{\mathrm{d}v}{\mathrm{d}x} = 5\sqrt{3} - 1.5 - 3x$, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P at a displacement x m from O. [3]

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(ii) Find the value of x for which P reaches its maximum velocity, and calculate this maximum velocity. [4]

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(iii) Calculate the magnitude of the acceleration of P immediately after it has first come to instantaneous rest. [4]

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Q23

1 A particle P of mass 0.2 kg is released from rest at a point O on a smooth horizontal surface. A horizontal force of magnitude te^{-v} N directed away from O acts on P, where v m s^{-1} is the velocity of P at time t s after release. Find the velocity of P when $t = 2$. [4]

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Q24

3 A particle P of mass 0.4 kg is released from rest at a point O on a smooth plane inclined at 30° to the horizontal. P moves down the line of greatest slope through O. The velocity of P is v m s^{-1} when its displacement from O is x m. A retarding force of magnitude $0.2v^2$ N acts on P in the direction PO.

(i) Show that $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 5 - 0.5v^2$. [2]

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(ii) Express v in terms of x. [4]

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6 A particle P of mass 0.2 kg is released from rest at a point O on a plane inclined at 30° to the horizontal. At time t s after its release, P has velocity v m s^{-1} and displacement x m down the plane from O. The coefficient of friction between P and the plane increases as P moves down the plane, and equals $0.1x^2$.

(i) Show that $2v\frac{dv}{dx} = 10 - \sqrt{3})x^2$. [2]

(ii) Calculate the maximum speed of P. [5]

(iii) Find the value of x at the point where P comes to rest. [2]

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Q26

3 A particle P of mass 0.4 kg is released from rest at a point O on a smooth plane inclined at 30° to the horizontal. When the displacement of P from O is x m down the plane, the velocity of P is v m s^{-1}. A force of magnitude $0.8\mathrm{e}^{-x}$ N acts on P up the plane along the line of greatest slope through O.

(i) Show that $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 5 - 2\mathrm{e}^{-x}$. [2]

(ii) Find v when $x = 0.6$. [4]

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[Turn over

5 A particle P of mass 0.4 kg is placed at rest at a point A on a rough horizontal surface. A horizontal force, directed away from A and with magnitude $0.6t$ N, acts on P, where t s is the time after P is placed at A. The coefficient of friction between P and the surface is 0.3, and P has displacement from A of x m at time t s.

(i) Show that P starts to move when $t = 2$. Show also that when P is in motion it has acceleration $1.5t - 3)\,\text{m s}^{-2}$. [3]

(ii) Express the velocity of P in terms of t, for $t \geqslant 2$. [4]

(iii) Express x in terms of t, for $t \geqslant 2$. [3]

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3 A small block B of mass 0.25 kg is released from rest at a point O on a smooth horizontal surface. After its release the velocity of B is v m s^{-1} when its displacement is x m from O. The force acting on B has magnitude $2 + 0.3x^2$) N and is directed horizontally away from O.

(i) Show that $v\frac{\mathrm{d}v}{\mathrm{d}x} = 1.2x^2 + 8$. [2]

(ii) Find the velocity of B when $x = 1.5$. [3]

An extra force acts on B after $x = 1.5$. It is given that, when $x > 1.5$,

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = 1.2x^2 + 6 - 3x.$$

(iii) Find the magnitude of this extra force and state the direction in which it acts. [2]

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Q29

5 A particle P of mass 0.4 kg is released from rest at a point O on a smooth plane inclined at 30° to the horizontal. A force of magnitude $3e^{-t}$ N directed up a line of greatest slope acts on P, where t s is the time after release.

(i) Show that $\frac{dv}{dt} = 7.5e^{-t} - 5$, where v m s^{-1} is the velocity of P up the plane at time t s. [2]

(ii) Express v in terms of t. [3]

(iii) Find the distance of P from O when v has its maximum value. [3]

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Linear Motion Under Variable Force MS

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Q1

Question	Answer	Marks	Guidance
1	$\frac{\mathrm{d}v}{\sqrt{v}} = -\frac{10\mathrm{d}t}{(t+1)^2}$	**M1**	Separate variables.
	$2\sqrt{v} = \frac{10}{t+1} + A$	**M1 A1**	Attempt to integrate.
	$t = 0, \quad v = 25, \quad A = 0$	**M1**	Use correct initial condition.
	$v = \frac{25}{(t+1)^2}$	**A1**	CAO
		5	

Q2

Question	Answer	Marks	Guidance
5(a)	$m\frac{dv}{dt} = -mg - 2mv$	**B1**	Use of SUVAT implies 0 marks. N2L, must include m .
	$\ln(5+v) = -2t(+A)$	**M1**	Separate variables and integrate 3-term N2L, condone omission of constant.
	$\ln(5+v) = -2t + A$	**A1 FT**	FT only sign error in N2L.
	$t = 0, v = 20, \quad A = \ln 25$	**M1**	Use correct initial condition.
	$2t = \ln\left(\frac{25}{5+v}\right)$, $e^{2t} = \frac{25}{5+v}$	**M1**	Remove all logs.
	$v = 25e^{-2t} - 5$	**A1**	
	Alternative method for question 5(a)		
	$m\frac{dv}{dt} = -mg - 2mv$	**B1**	N2L, must include m .
	$\frac{dv}{dt} + 2v = -g$: Integrating factor = e^{2t}	**M1**	
	$\frac{d\left(ve^{2t}\right)}{dt} = -ge^{2t}, \quad ve^{2t} = -\frac{g}{2}e^{2t}(+A)$	**M1**	Integrate both sides, condone omission of constant.
	$ve^{2t} = -\frac{g}{2}e^{2t} + \mathrm{A}$	**A1 FT**	FT only sign error in N2L.
5(a)	$t = 0, v = 20, \quad A = 25$	**M1**	Use correct initial condition.
	$ve^{2t} = -\frac{g}{2}e^{2t} + 25, \quad v = 25e^{-2t} - 5$	**A1**	
		6	
5(b)	$x = -\frac{25}{2}e^{-2t} - 5t(+B)$	**M1**	Use of SUVAT implies 0 marks. Integrate their expression from part **(a)**.
	$t = 0, x = 0, \quad B = \frac{25}{2}$ $x = \frac{25}{2}\left(1 - e^{-2t}\right) - 5t$	**A1 FT**	FT only expressions of the form $v = Pe^{kt} + Q$ for P, Q non-zero.
		2	
5(c)	Greatest height when $v = 0$, so $t = 0.8047\ldots$ or $\frac{1}{2}\ln 5$	**M1**	Use of SUVAT in part **(a)** or part **(b)** implies 0 marks. Find value of t, may be embedded.
	$x = 5.98$ m	**A1**	CWO
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Q3

Question	Answer	Marks	Guidance
2(a)	Separate variables and integrate: $\frac{dv}{v}=\left(\frac{1-2t^2}{t}\right)dt$ so $\ln\lvert v\rvert=\ln t-t^2+c$	**M1 A1**	Must include logs. Condone missing modulus.
	$\lvert v\rvert=Ate^{-t^2}$, $-v=Ate^{-t^2}$, $v=-Ate^{-t^2}$	**A1**	CAO.
		3	
2(b)	$a=\frac{-Ate^{-t^2}\left(1-2t^2\right)}{t}=-Ae^{-t^2}\left(1-2t^2\right)$	**M1**	Substituting their answer to part **(a)** into given formula
	$t=1,\ a=5\quad(A=5e)$	**M1**	Use initial condition.
	Force = $5me^{1-t^2}\left(2t^2-1\right)$	**A1**	Use N2L, correct work only.
	Alternative method for question 2(b)		
	$a=\frac{v\left(1-2t^2\right)}{t}$ substitute $t=1,\ a=5$ so $v=-5$	**M1**	Use initial condition. Use N2L, correct work only.
	Substituting in their answer to part **(a)** so $(A=5e)$	**M1**	
	Force = $5me^{1-t^2}\left(2t^2-1\right)$	**A1**	
		3	

Q4

Question	Answer	Marks	Guidance
6(a)	$2v\frac{dv}{dx}=8x-\frac{128}{x^3}$	**M1**	Separate variables and integrate, + c not required for M1.
	$v^2=4x^2+64x^{-2}+c$	**A1**	OE.
	$x=8,\ v=-15$ and $c=-32$	**M1**	Use initial condition.
	$v^2=\frac{4}{x^2}\left(x^4-8x^2+16\right)$ or $4x^2-32+\frac{64}{x^2}$	**A1**	Correct expression for v^2, AEF.
	$v^2=\frac{4}{x^2}\left(x^2-4\right)^2$ giving $v=-\frac{2}{x}\left(x^2-4\right)$	**A1**	Convincingly shown, e.g. v is negative initially, AG.
		5	
6(b)	$\frac{1}{2}\ln(x^2-4)=-2t(+A)$	**M1**	Use $v=\frac{dx}{dt}$ and integrate.
	$t=0,\ x=8,\ \ A=\frac{1}{2}\ln 60$	**DM1**	Use initial condition.
	$\frac{1}{2}\ln\left(\frac{x^2-4}{60}\right)=-2t$ giving $\frac{x^2-4}{60}=e^{-4t}$	**M1**	Remove log.
	$x=\sqrt{4+60e^{-4t}}$	**A1**	CAO
		4	

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Q5

Question	Answer	Marks
5(a)	$\frac{\mathrm{d}v}{3u-v}=k\mathrm{d}t$	**M1**
	$-\ln(3u-v)=kt+d$ $t=0, v=u:\quad d=-\ln 2u$	**M1**
	$v=2u:\quad t=\frac{1}{k}\ln 2$	**A1**
		3
5(b)	$v\frac{\mathrm{d}v}{\mathrm{d}x}=3ku-kv\ [\Rightarrow \frac{v\mathrm{d}v}{3u-v}=k\mathrm{d}x]$	**B1**
	$\frac{(-(3u-v)+3u)\mathrm{d}v}{3u-v}=k\mathrm{d}x$ so $-v-3u\ln(3u-v)=kx+c$	**M1A1**
	$x=0, v=u:\ c=-u-3u\ln 2u$	**M1**
	$v=2u:\ x=\frac{u}{k}(3\ln 2-1)$	**A1**
		5

Q6

Question	Answer	Marks
2	$\frac{\mathrm{d}v}{g-kv}=\mathrm{d}t$	**M1**
	$-\frac{1}{k}\ln(g-kv)=t+A$	**M1**
	$t=0, v=0:\ A=-\frac{1}{k}\ln g$	**A1**
	$t=-\frac{1}{k}\ln\frac{g-kv}{g}$	**M1**
	$v=\frac{g}{k}(1-e^{-kt})$	**M1A1**
		6

Q7

Question	Answer	Marks	Guidance
7(a)	$v\frac{\mathrm{d}v}{\mathrm{d}x}=-\frac{100}{x^3}+\frac{200}{x^2}$ $\frac{v^2}{2}=\frac{50}{x^2}-\frac{200}{x}+A$	**M1 A1**	Correct equation and attempt to integrate Correct
	$x=1, v=-10:\quad A=200$	**M1**	Use initial condition
	$v^2=\frac{100(2x-1)^2}{x^2}$	**M1**	Rearrange to find v^2
	$v=\pm\frac{10(2x-1)}{x}$ and take negative sign to meet initial condition, so $v=\frac{10(1-2x)}{x}$	**A1**	Convincingly shown (no mention of ± scores A0) AG
		5	

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7(b)	$\frac{x\mathrm{d}x}{1-2x}=10\mathrm{d}t$ $\frac{1}{2}\left(\frac{1}{1-2x}-1\right)\mathrm{d}x=10\mathrm{d}t$ $-\frac{1}{4}\ln\lvert1-2x\rvert-\frac{x}{2}=10t+B$	**M1 A1**	Rearrange and attempt to integrate
	$t=0,x=1:\quad B=-\frac{1}{2}$	**M1**	Use initial condition
	$2x-2=-40t-\ln(\lvert1-2x\rvert)$ so $e^{-40t}=(2x-1)e^{2x-2}$	**A1**	Convincingly shown, working required AG
	For large values of t, $x\to\frac{1}{2}$	**B1**	CAO
		5	

Q8

Question	Answer	Marks	Guidance
7(a)	$mv\frac{dv}{dx}=-kmv^2$	**B1**	N2L, with m
	$\ln v=-kx+c$	**M1**	Separate variables and integrate
	$x=0,v=u:\quad c=\ln u$	**M1**	Use initial condition
	$v=\frac{1}{2}u:\quad \ln\frac{1}{2}=-kx,$ $x=\frac{1}{k}\ln 2$	**A1**	AG
		4	
7(b)	$mv\frac{\mathrm{d}v}{\mathrm{d}x}=-mkv^2+\frac{5m}{v}$	**B1**	N2L (allow missing m in this part)
	$\frac{v^2\mathrm{d}v}{5-kv^3}=\mathrm{d}x\ -\frac{1}{3k}\ln\left(5-kv^3\right)=x(+d)$	**M1A1**	Separate variables and integrate
	Using **(a)** $-\frac{1}{3k}\ln\left(5-kv^3\right)=x-\frac{1}{3k}\ln\left(5-ku^3\right)-\frac{1}{k}\ln 2$	**M1M1**	Use condition. M0 if $v=\frac{1}{2}u,x=0$ used unless $\frac{1}{k}\ln 2$ is added on later Rearrange dependent on ln solution
	$x=\frac{1}{3k}\ln\left(\frac{40-ku^3}{5-ku^3}\right)$	**M1A1**	Use $v=u$
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Q9

Question	Answer	Marks	Guidance
7(i)	$0.2mg = 0.06 \times 8$	**M1**	Resolve along the plane
	$m = 0.24$ kg **AG**	**A1**	
		2	
7(ii)	$m\frac{dv}{dt} = 0.06t - 0.2mg$ or $0.24\frac{dv}{dt} = 0.06t - 0.2 \times 0.24g$	**M1**	Use N2L along the plane
	$\frac{dv}{dt} = 0.25t - 2$ **AG**	**A1**	
	$\int dv = \int (0.25t - 2)dt$	**M1**	Attempt to integrate
	$v = 0.25t^2 / 2 - 2t + c$, Put $v = 0$ and $t = 4$ (leads to $c = 6$)	**M1**	Attempt to find c
	Initial velocity = 6 m s^{-1}	**A1**	
		5	
7(iii)	$x = \int (0.25t^2/2 - 2t + 6)dt$	**M1**	Attempt to integrate
	$x = 0.25t^3/6 - t^2 + 6t$ (+ k)	**A1ft**	ft candidates c from part **(ii)**
	Finds or assumes $k = 0$ and substitutes $t = 4$ OR uses limits of 0 and 4	**M1**	
	OP = 32/3 = $10\frac{2}{3}$ = 10.7 m	**A1**	
		4	

Q10

Question	Answer	Marks	Guidance
7(i)	$0.5v\frac{dv}{dx} = -0.5g - 0.1x^2$	**M1**	Use Newton's Second Law vertically
	$v\frac{dv}{dx} = -10 - 0.2x^2$ **AG**	**A1**	
		2	
7(ii)	$\int v dv = \int (-10 - 0.2x^2)dx$	**M1**	Attempt to integrate the expression in part **(i)**
	$\frac{v^2}{2} - 10 \times \frac{-0.2x^3}{3} + c$	**A1**	
	$\left[\frac{v2}{2} = -10 - \frac{0.2}{3} + 18\right]$	**M1**	Either use limits or find c and put $x = 1$
	$v = 3.98$ (329…) ms^{-1}		
7(iii)	$0.5v\frac{dv}{dx} = -0.5g - 0.1x^2 - \frac{16(x-1)}{1}$	**M1**	Use Newton's Second Law vertically when string becomes taut
	$v\frac{dv}{dx} = -10 - 0.2x^2 - 32x + 32 = 22 - 32x - 0.2x^2$	**A1**	
		2	
7(iv)	$\int v dv = \int (22 - 32x - 0.2x^2)dx$	**M1**	Attempt to integrate after the string becomes taut
	$\frac{v^2}{2} = 22x - \frac{32x^2}{2} - \frac{0.2x^3}{3} + k$	**A1**	
	$x = 1$, $v = 3.98$ (329…) hence $k = 2$. Now put $x = 1.5$ $22 \times 1.5 - 32 \times \frac{1.5^2}{2} - 0.2 \times \frac{1.5^3}{3} + 2 = -1.225$	**M1**	Either use limits or find k and put $x = 1.5$
	As $\frac{v^2}{2}$ cannot be negative, P comes to rest before the extension of the string is 0.5.	**A1**	
		4	

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Q11

Question	Answer	Marks	Guidance
4(i)	$0.5v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 0.5g - \dfrac{16x}{0.8} - 25x^2$	**M1**	Use Newton's Second Law vertically
	$v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 10 - 40x - 50x^2$ **AG**	**A1**	
		2	
4(ii)	$\int v\mathrm{d}v = \int\left(10 - 40x - 50x^2\right)\mathrm{d}x$	**M1**	Attempt to integrate
	$\dfrac{v^2}{2} = 10x - 20x^2 - \dfrac{50x^3}{3}(+c)$	**A1**	
	$0 = 10 - 40x - 50x^2$	**M1**	Put the acceleration equal to zero
	$x = 0.2$ (Ignore $x = -1$ if seen)	**A1**	
	$\dfrac{0.5v^2}{2} = \dfrac{8}{15} = 0.533\,\mathrm{J}$	**B1**	Use $KE = \dfrac{mv^2}{2}$
	$16 \times \dfrac{0.2^2}{(2 \times 0.8)} = 0.4\,\mathrm{J}$	**B1**	Use $EE = \dfrac{\lambda x^2}{(2l)}$
		6	

Q12

Question	Answer	Marks	Guidance
3(i)	$0.25v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -kv^2x^{-2} \rightarrow v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -4kv^2x^{-2}$	**B1**	AG
		1	
3(ii)	$\int\dfrac{\mathrm{d}v}{v} = -4k\int x^{-2}\mathrm{d}x$	**M1**	Attempt to integrate
	$\ln v = \dfrac{4k}{x}(+c)$	**A1**	
	$x = 0.8$, $v = 3$ hence $c = \ln 3 - 5k$	**A1**	Finds c
	$\ln v = \dfrac{4k}{x} + \ln 3 - 5k$	**M1**	
	$v = 3^{\left(\frac{4k}{x} - 5k\right)}$	**A1**	
		5	

Q13

Question	Answer	Marks	Guidance
6(i)	$0.2v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 0.09\sqrt{x} - 0.3$	**M1**	Use Newton's Second Law horizontally
	$v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 0.45\sqrt{x} - 1.5$	**A1**	AG
		2	
6(ii)	$0 = 0.45\,x^{\frac{1}{2}} - 1.5$	**M1**	Equate acceleration to zero
	$x = \dfrac{100}{9}$	**A1**	
		2	

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6(iii)	$\int v\,\mathrm{d}v = \int (0.45x^{\frac{1}{2}} - 1.5)\mathrm{d}x$	**M1**	Attempt to integrate
	$\frac{v^2}{2} = \frac{0.45^{\frac{3}{2}}}{\frac{3}{2}} - 1.5x(+c) = 0.3x^{\frac{3}{2}} - 1.5x(+c)$	**A1**	
	$0.3\left(\frac{100}{9}\right)^{\frac{3}{2}} - 1.5\left(\frac{100}{9}\right) + c = 0$	**M1**	
	$c = \frac{50}{9}$	**A1**	
	$x = 0,\ \frac{v^2}{2} > \frac{50}{9}$ so $v > \frac{10}{3}$	**A1**	
		5	

Q14

Question	Answer	Marks	Guidance
6(i)	$R = 0.2g + 0.4t\sin\theta$ ($= 2 + 0.24t$) $F = 0.5(2 + 0.24t) = 1 + 0.12t$	**M1**	Note $\sin\theta = 0.6$ and $\cos\theta = 0.8$ ($\theta = 36.87°$) Resolve vertically and use $F = \mu R$
	$0.4t\cos\theta = 1 + 0.12t$	**M1**	Resolve horizontally
	$t = 5$	**A1**	
		3	
6(ii)	$v/\mathrm{d}t = 0.4t \times 0.8 - (1 + 0.12t)$	**M1**	Use Newton's Second Law horizontally
	$\mathrm{d}v / \mathrm{d}t = t - 5$ **AG**	**A1**	
		2	
6(iii)	$\int \mathrm{d}v = \int (t-5)\mathrm{d}t$ $v = t^2/2 - 5t + c$	**M1**	Attempt to integrate the equation from part **(ii)**
	$v = 0$ when $t = 5$ hence $c = 12.5$	**A1**	Finds the constant of integration, c
	$v = 8^2/2 - 5 \times 8 + 12.5 = 4.5$	**A1**	Find v when $t = 8$
	$a = -0.5 \times 0.2g / 0.2 = -5\ \mathrm{m\,s^{-1}}$ and $s = 4.5^2/(2 \times 5)$	**M1**	Finds a and uses $v^2 = u^2 + 2as$
	$s = 2.025$ m	**A1**	
		5	

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Q15

Question	Answer	Marks	Guidance
3(i)	$0.4\frac{dv}{dt} = 0.8t - 2e^{-t}$	**M1**	Use Newton's Second Law horizontally
	$\frac{dv}{dt} = 2t - 5e^{-t}$	**A1**	**AG**
		2	
3(ii)	$\int dv = \int(2t - 5e^{-t})dt$ $v = t^2 + 5e^{-t}(+ c)$	**M1**	Attempt to integrate the equation from part **(i)**
	$t = 1$ and $v = 8$ so c = 5.16	**M1**	Attempt to find the constant of integration, c
	$v = t^2 + 5e^{-t} + 5.16$ or $v = t^2 + 5e^{-t} + 7 - 5e^{-1}$	**A1**	
		3	
3(iii)	Eval v for $t = 0$	**M1**	
	$V = 10.2$ ms^{-1}	**A1**	
		2	

Q16

Question	Answer	Marks	Guidance
7(i)	$0.2dv/dt = 0.2\,g + 0.6t - k\,e^{-t}$	**M1**	Use Newton's Second Law downwards
	$dv/dt = 10 + 3t - 5\,ke^{-t}$ **AG**	**A1**	
	Total:	**2**	
7(ii)	$dv/dt = 10 - 5k\,e^{0} = 0$	**M1**	Recognise that dv/dt = 0 when t = 0
		M1	Attempts to solve the equation
7(ii)	k = 2	**A1**	
	Total:	**3**	
7(iii)	$\int dv = \int(10 + 3t - 5k\,e^{-t})dt$	**M1**	Attempts to integrate the equation from part i with k not replaced
	[v = $10t + 3t^2/2 + 5e^{-t} + c$, v = 0, t = 0 so c = – 5] v = $10t + 3t^2/2 + 5e^{-t} - 5$	**A1**	
	$\int dx = \int(10t + 3t^2/2 + 5e^{-t} - 5)dt$ $x = 5t^2 + t^3/2 - 5e^{-t} - 5t + c$	**M1**	Attempts to integrate again. Allow their k or just k not replaced
	$x = 0$, t = 0, so c = 5 and substitutes t = 2 $x = 5 \times 2^2 + 2^3/2 - 5e^{-2} - 5 \times 2 + 5$	**M1**	
	Height = 18.3 m	**A1**	
	Total:	**5**	

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Q17

Question	Answer	Marks	Guidance
4(i)	$T = 16(1.6 - 0.8 - x)/0.8$ ($= 16 - 20x$)	**B1**	Use $T = \lambda x/L$
	$0.5v\mathrm{d}v/\mathrm{d}x = 16(1.6 - 0.8 - x)/0.8 - 48x^2$	**M1**	Use Newton's Second Law horizontally
	$v\mathrm{d}v/\mathrm{d}x = 32 - 40x - 48x^2$ **AG**	**A1**	
		3	
4(ii)	$x^2 + 40x - 32 = 0$	**M1**	Put acceleration = 0 for maximum velocity
	$x = 0.5$	**A1**	
	$\int v\mathrm{d}v = \int (32 - 40x - 48x^2)\mathrm{d}x$ $(v^2/2 = 32x - 40x^2/2 - 48x^3/3 + c)$	**M1**	Attempt to integrate the equation from part **(i)**
	$4.5^2/2 = 32\times0.5 - 20\times0.5^2 - 16\times0.5^3 + c$, $c = 1.125$	**M1**	Substitute $x = 0.5$, $v = 4.5$ to find c
	$v = 1.5$	**A1**	Use $x = 0$
		5	

Q18

Question	Answer	Marks	Guidance
3(i)	a ($= 0.4v\mathrm{d}v/\mathrm{d}x$) $= 8x - (2e^{-x} + 4)$	**M1**	Use Newton's Second Law horizontally
	$v\mathrm{d}v/\mathrm{d}x = 20x - 10 - 5e^{-x}$	**A1**	
		2	
3(ii)	$\int v\mathrm{d}v = \int (20x - 10 - 5e^{-x})\,\mathrm{d}x$	**M1**	Attempt to integrate the equation from part **(i)**
	$\left[v^2/2\right]_6^v = \left[10x^2 - 10x + 5e^{-x}\right]_0^{0.5}$	**M1**	Use correct limits in the integration
	v = 5.2(0)	**A1**	
		3	
3(iii)	$x - 4 = T = \lambda(x - 0.5)/0.5$ ($= \lambda x/0.5 - \lambda$)	**M1**	Use $T = \lambda x/L$
	$\lambda = 4$ N	**A1**	
		2	

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Q19

Question	Answer	Marks	Guidance
6(i)	Friction = 0.4 x 0.4g	**B1**	Uses $F = \mu R$
	$0.4v\mathrm{d}v/\mathrm{d}x = -0.4$ x $0.4g - 0.8/x$	**M1**	Uses Newton's Second Law and $a = v\mathrm{d}v/\mathrm{d}x$
	$v\mathrm{d}v/\mathrm{d}x = -4 - 2/x$	**A1**	
	Total:	**3**	
6(ii)	$\int v\mathrm{d}v = \int\left(-4 - \frac{2}{x}\right)\mathrm{d}x$	**M1**	Separates the variables and attempts to integrate
	$v^2/2 = -4x - 2\ln x$ ($+ c$)	**A1**	
	$v = U$ when $x = 1$ hence $c = U^2/2 + 4$	**M1**	Attempts to find c
	$0 = -4$ x $2 - 2\ln 2 + U^2/2 + 4$ [$U^2 = 10.7(725)$] $0 = -4$ x $2.1 - 2\ln 2.1 + U^2/2 + 4$ [$U^2 = 11.7(677)$]	**M1**	Put $v = 0$ and $x = 2$ Put $v = 0$ and $x = 2.1$
	$3.28 < U < 3.43$	**A1**	
	Total:	**5**	

Q20

7(i)	$\mu = 0.6 \times 0.5^2 \ / (0.5\,g)\ (= 0.03)$	**B1**	Uses $F = \mu R$
	$0.5\mathrm{d}v / \mathrm{d}t = 0.6t^2 - 0.03 \times 0.5g$	**M1**	Uses Newton's Second Law horizontally
	$\mathrm{d}v / \mathrm{d}t = 1.2t^2 - 0.3$	**A1**	
	Total:	**3**	
7(ii)	$\int \mathrm{d}v = \int (1.2t^2 - 0.3)\,\mathrm{d}t$ $v = 0.4t^3 - 0.3t\ (+ \mathrm{c})$	**M1**	Separates the variables and attempts to integrate
	$t = 0.5,\ v = 0$ hence c = 0.1	**M1**	Attempts to find c
	$v = 0.4t^3 - 0.3t + 0.1$	**A1**	
	Total:	**3**	
7(iii)	$\int \mathrm{d}x = \int (0.4t^3 - 0.3\mathrm{t} + 0.1)\,\mathrm{d}t$ $x = 0.1t^4 - 0.15t^2 + 0.1t\ (+ \mathrm{c})$	**M1**	Attempts to integrate
	$t = 0.5,\ x = 0$ hence c = –0.01875	**M1**	Finds c or substitutes the limits
	$x(1.2) = 0.0926(1)$	**A1**	
	Total:	**3**	

Q21

4(i)	$0.4a = 0.4g - 0.2v^2$	**M1**	Uses Newton's Second Law vertically
	$v\mathrm{d}v / \mathrm{d}x = 10 - 0.5v^2$	**A1**	AG
	Total:	**2**	
4(ii)	$\int v\ \mathrm{d}v / (10 - 0.5v^2) = \int \mathrm{d}x$	**M1**	Separates the variables and attempts to integrate
	$-\ln(10 - 0.5v^2) = x\ (+ \mathrm{c})$	**A1**	
	$x = 0,\ v = 0$ hence c = –ln10	**M1**	Attempts to find c using $x = 0,\ v = 0$
	$v = \sqrt{(20 - 20\mathrm{e}^{-x})}$	**A1**	$10–0.5v^2 = \mathrm{e}^{-x+ln10} = 10\,\mathrm{e}^{-x}$
	Total:	**4**	
4(iii)	Increase$= \sqrt{(20 - 20\mathrm{e}^{-8})} - \sqrt{(20 - 20\mathrm{e}^{-4})}$	**M1**	**M1** if x values are substituted into their value for part (ii)
	Increase = 0.0404 m s^{-1}	**A1**	Allow 0.04
	Total:	**2**	

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Q22

Question	Answer	Marks	Guidance
7(i)	$v\mathrm{d}v/\mathrm{d}x = 0.2g\sin 60 - 0.3 \times 0.2g\cos 60 - 0.6x$	**M1A1**	Uses Newton's Second Law parallel to the plane. Correct equation.
	$v\mathrm{d}v/\mathrm{d}x = 5\sqrt{3} - 1.5 - 3x$	**A1**	AG
		3	
7(ii)	$x = (5\sqrt{3} - 1.5)/3$ (= 2.39)	**B1**	Uses $a = 0$.
	$\int v\,\mathrm{d}v = \int(5\sqrt{3} - 1.5 - 3x)\,\mathrm{d}x$	**M1**	Separates the variables and attempts to integrate.
	$v^2/2 = 5\sqrt{3}\,x - 1.5x - 3x^2/2\ (+c)$	**A1**	Allow $c = 0$ without calculation seen.
	$v = 4.13$	**A1**	Substitutes $x = 2.39$.
		4	
7(iii)	$0 = 5\sqrt{3}\,x - 1.5x - 3x^2/2$	**M1**	Puts $v = 0$ and attempts to solve a quadratic equation.
	$x = 4.77(35...)$	**A1**	
	$a = 5\sqrt{3} - 1.5 - 3 \times 4.77(35\ldots)$	**M1**	
	Magnitude of $a = 7.16\ \mathrm{m\,s^{-2}}$	**A1**	
		4	

Q23

Question	Answer	Marks	Guidance
1	$0.2\mathrm{d}v/\mathrm{d}t = t\,\mathrm{e}^{-v}$	**M1**	Uses Newton's Second Law to set up a differential equation. Allow a for $\mathrm{d}v/\mathrm{d}t$.
	$\int \mathrm{e}^v\,\mathrm{d}v = 5\int t\,\mathrm{d}t$ leading to $\mathrm{e}^v = 5t^2/2\ (+c)$	**M1**	Separates the variables and integrates.
	$\mathrm{e}^v - 1 = 2.5t^2$	**A1**	Substitutes $t = 0$, $v = 0$.
	$v(2) = \ln 11 = 2.4$	**A1**	
		4	

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Q24

Question	Answer	Marks	Guidance
3(i)	$0.4v\mathrm{d}v/\mathrm{d}x = 0.4g\sin30 - 0.2v^2$	**M1**	Uses Newton's Second Law down the plane. Allow a for $v\mathrm{d}v/\mathrm{d}x$.
	$v\mathrm{d}v/\mathrm{d}x = 5 - 0.5v^2$	**A1**	AG
		2	
3(ii)	$\int v/(5-0.5v^2)\mathrm{d}v = \int x\ \mathrm{d}x$	**M1**	Separates the variables and attempts to integrate.
	$-\ln(5-0.5v^2) = x\ (+c)$	**A1**	
	$c = -\ln5\ [5-0.5v^2 = 5\mathrm{e}^{-x}]$	**M1**	Puts $x = 0$, $v = 0$ to find c and attempts to solve for v.
	$v = \sqrt{(10-10\mathrm{e}^{-x})}$	**A1**	
		4	

Q25

6 (i)	$0.2v\mathrm{d}v/\mathrm{d}x = 0.2g\sin30 - 0.1x^2(0.2g\cos30)$	**M1**		N2L parallel to the slope
	$2v\mathrm{d}v/\mathrm{d}x = 10 - (\sqrt{3})x^2$ AG	**A1**	2	
(ii)	$2\int v\mathrm{d}v = \int(10-\sqrt{3}x^2)\ \mathrm{d}x$	**M1**		Integrates acceleration
	$v^2 = 10x - \sqrt{3}x^3/3$	**A1**		No need to show c = 0
	$0 = 10 - \sqrt{3}x^2$	**M1***		Solves accn =0 ($x = 2.4028..$)
	$v^2 = 10 \times 2.4 - \sqrt{3} \times 2.4^3/3$	**dep M1***		Puts solution of accn = 0 in v^2 (x)
	Max $v = 4(.002)$	**A1**	5	
(iii)	$0 = 10x - \sqrt{3}x^3/3$	**M1**		
	$x = 4.16$	**A1**	2	

Q26

3 (i)	$0.4v\mathrm{d}v/\mathrm{d}x = 0.4\,g\sin30 - 0.8e^{-x}$	**M1**		
	$v\mathrm{d}v/\mathrm{d}x = 5 - 2e^{-x}$ **AG**	**A1**	2	
(ii)	$\int v\mathrm{d}v = \int(5-2e^{-x})\mathrm{d}x$	**M1**		Separates the variables and attempts to integrate
	$v^2/2 = 5x + 2e^{-x}$ (+	**A1**		
		M1		Uses limits or finds c (c = –2)
	$v = 2.05$	**A1**	4	

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Q27

5 (i)	$t = 2$ **AG**	**B1**		From $0.6t = 0.3 \times 4$ or $1.5t - 3 = 0$
	$0.4a = 0.6t - 0.3 \times 0.4\,g$	**M1**		
	$a = 1.5t - 3$ **AG**	**A1**	3	
(ii)	$v = \int(1.5t - 3)\,dt$	**M1**		
	$v = 0.75t^2 - 3t\ (+ c)$	**A1**		
	$0 = 0.75 \times 2^2 - 3 \times 2 + c$ (so c=3)	**M1**		Or uses limits with 2 and t
	$v = 0.75t^2 - 3t + 3$	**A1**	4	
(iii)	$x = \int(0.75t^2 - 3t + 3)dt$	**M1**		
	$x = 0.25t^3 - 1.5t^2 + 3t\ (+ k)$	**A1**√		ft c from (ii)
	$x = 0.25t^3 - 1.5t^2 + 3t - 2$	**A1**	3	

Q28

3 (i)	$0.25v\,dv/dx = 2 + 0.3x^2$	**M1**		
	$v\,dv/dx = 1.2x^2 + 8$ AG	**A1**	2	
(ii)	$\int v\ dv = \int(1.2x^2 + 8)\ dx$	**M1**		
	$v^2/2 = 0.4x^3 + 8x\ (+ c)$	**A1**		Allow c = 0 without working
	v = 5.17	**A1**	3	
(iii)	$0.25v\,dv/dx = 0.3x^2 + 1.5 - 0.75x$	**M1**		
	Force is $0.5 + 0.75x$ N towards O	**A1**	2	

Q29

5 (i)	$0.4\,dv/dt = 3e^{-t} - 0.4g\sin 30$	**M1**		
	$dv/dt = 7.5e^{-t} - 5$ AG	**A1**	[2]	
(ii)		**M1**		Integrates accn
	$\int_0^v dv = \int_0^t (7.5e^{-t} - 5)\,dt$	**M1**		Limits or finds integration constant
	$v = 7.5 - 7.5e^{-t} - 5t$	**A1**	[3]	
(iii)	Solves $dv/dt = 0$	**M1**		$t = 0.405(46\ldots)$
	$\int_0^x dx = \int_0^{0.405} (7.5 - 7.5e^{-t} - 5t)\,dt$	**M1**		Integrates expression for v and uses $t = 0.405$
	$x = 0.13(0)$ m	**A1**	[3]	

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Momentum

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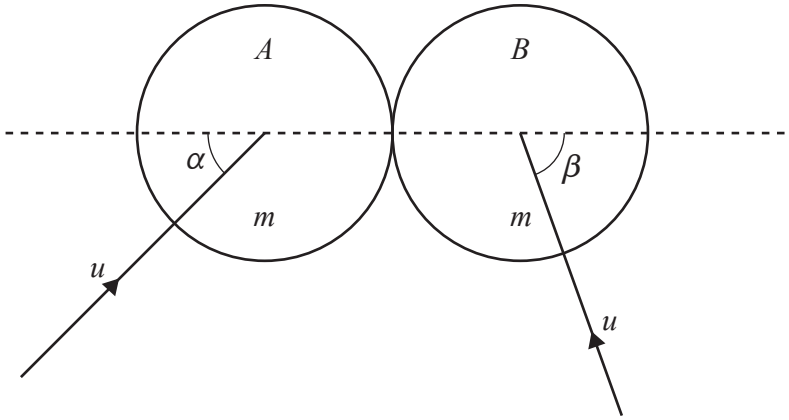
6



Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle α with the line of centres, and B's direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2\cos\beta = \cos\alpha$.

(a) Show that the direction of motion of A after the collision is perpendicular to the line of centres. [4]

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The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

(b) Find the value of α. [4]

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Q2

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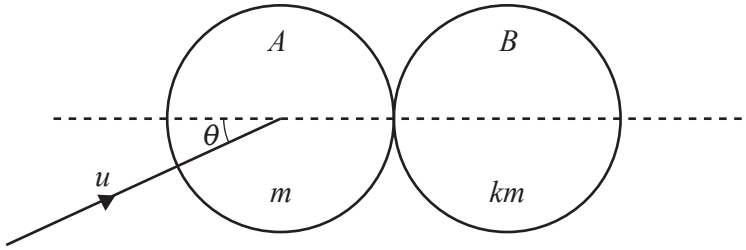

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

(a) Show that the speed of B after the collision is $\dfrac{4u\cos\theta}{3(1+k)}$. [3]

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70% of the total kinetic energy of the spheres is lost as a result of the collision.

(b) Given that $\tan\theta = \frac{1}{3}$, find the value of k. [6]

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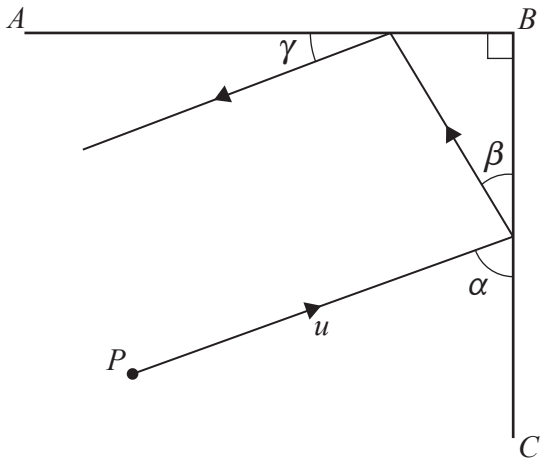

The smooth vertical walls AB and CB are at right angles to each other. A particle P is moving with speed u on a smooth horizontal floor and strikes the wall CB at an angle α. It rebounds at an angle β to the wall CB. The particle then strikes the wall AB and rebounds at an angle γ to that wall (see diagram). The coefficient of restitution between each wall and P is e.

(a) Show that $\tan\beta = e\tan\alpha$. [3]

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(b) Express γ in terms of α and explain what this result means about the final direction of motion of P. [4]

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As a result of the two impacts the particle loses $\frac{8}{9}$ of its initial kinetic energy.

(c) Given that $\alpha + \beta = 90°$, find the value of e and the value of $\tan\alpha$. [4]

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Q4

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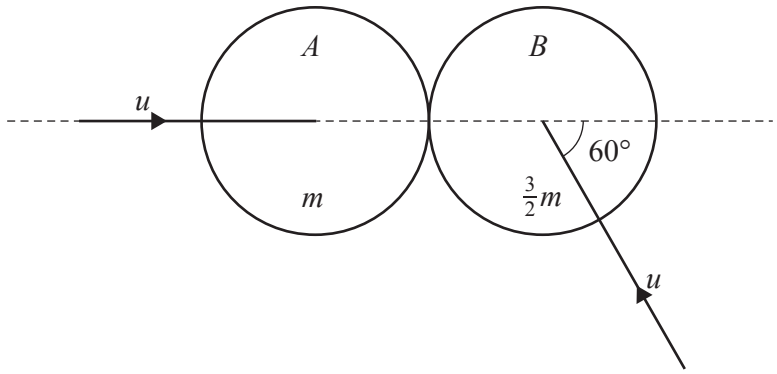

Two uniform smooth spheres A and B of equal radii have masses m and $\frac{3}{2}m$ respectively. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion is along the line of centres, and B's direction of motion makes an angle of 60° with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$.

(a) Find the angle through which the direction of motion of B is deflected by the collision. [6]

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(b) Find the loss in the total kinetic energy of the system as a result of the collision. [3]

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Q5

6 A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α with the barrier. The coefficient of restitution between P and the barrier is e. As a result of the impact, the direction of motion of P is turned through 90°.

(a) Show that $\tan^2\alpha = \frac{1}{e}$. [3]

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The particle P loses two-thirds of its kinetic energy in the impact.

(b) Find the value of α and the value of e. [5]

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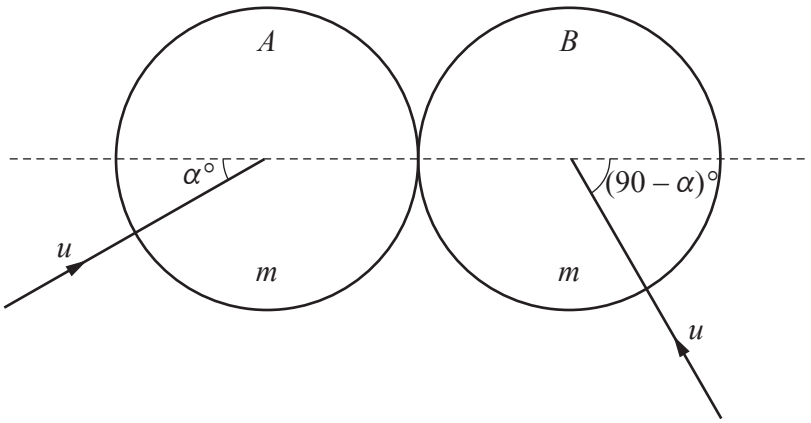

Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion makes an angle of $\alpha°$ with the line of centres, and B's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e.

Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan\alpha = \dfrac{1+e}{1-e}$. [4]

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(b) Given that $\tan\alpha = 2$, find the speed of A after the collision. [4]

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Q7

6 Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B. The coefficient of restitution between the spheres is e.

(a) Find, in terms of u and e, the velocities of A and B after the collision. [3]

Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B, where $\tan\theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B.

(b) Find the possible values of e. [7]

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Q8

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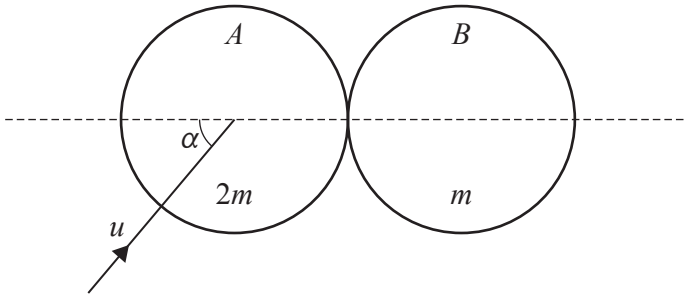

Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan\alpha = \frac{4}{3}$ (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

Find the speed of A after the collision. [5]

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3 Three uniform small spheres A, B and C have equal radii and masses $2m$, $4m$ and m respectively. The spheres are moving in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between each pair of spheres is e. Spheres A and B are moving towards each other with speeds $2u$ and u respectively. The first collision is between A and B.

(i) Find the velocities of A and B after this collision. [3]

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Sphere C is moving towards B with speed $\frac{4}{3}u$ and now collides with it. As a result of this collision, B is brought to rest.

(ii) Find the value of e. [4]

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(iii) Find the total kinetic energy lost by the three spheres as a result of the two collisions. [3]

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3 Three uniform small spheres A, B and C have equal radii and masses $3m$, m and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Find, in terms of u and e, expressions for the speeds of A, B and C after the first two collisions. [6]

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(ii) Given that A and C are moving with equal speeds after these two collisions, find the value of e. [3]

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Q11

3 Three uniform small spheres A, B and C have equal radii and masses $5m$, $5m$ and $3m$ respectively. The spheres are at rest on a smooth horizontal surface, in a straight line, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Show that the speed of A after its collision with B is $\frac{1}{2}u(1 - e)$ and find the speed of B. [3]

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Sphere B now collides with sphere C. Subsequently there are no further collisions between any of the spheres.

(ii) Find the set of possible values of e. [6]

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Q12

3 Two identical uniform small spheres A and B, each of mass m, are moving towards each other in a straight line on a smooth horizontal surface. Their speeds are u and ku respectively, and they collide directly. The coefficient of restitution between the spheres is e. Sphere B is brought to rest by the collision.

(i) Show that $e = \dfrac{k-1}{k+1}$. [3]

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(ii) Given that 60% of the total initial kinetic energy is lost in the collision, find the values of k and e. [6]

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2 Two uniform small spheres A and B have equal radii and masses $4m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e.

(i) Show that after the collision A moves with speed $\frac{1}{5}u(4 - e)$ and find the speed of B. [4]

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Sphere B continues to move until it collides with a fixed smooth vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{3}{4}e$. After this collision, the speeds of A and B are equal.

(ii) Find the value of e. [3]

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The spheres A and B now collide directly again.

(iii) Determine whether sphere B collides with the barrier for a second time. [2]

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2 Two uniform small smooth spheres A and B have equal radii and masses $5m$ and $2m$ respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is moving towards it with speed $2u$. The coefficient of restitution between the spheres is e.

(i) Show that the speed of B after the collision is $\frac{1}{7}u(1 + 15e)$ and find an expression for the speed of A. [4]

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In the collision, the speed of A is halved and its direction of motion is reversed.

(ii) Find the value of e. [2]

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(iii) For this collision, find the ratio of the loss of kinetic energy of A to the loss of kinetic energy of B. [3]

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2 Two uniform small smooth spheres A and B have equal radii and masses $2m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$.

(i) Find, in terms of u, the speeds of A and B after this collision. [4]

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Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is $\frac{1}{2}$.

(ii) Find, in terms of d and u, the time that elapses between the first and second collisions between A and B. [5]

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Q16

3 Two uniform small smooth spheres A and B have equal radii and masses $3m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e.

(i) Find, in terms of u and e, expressions for the velocities of A and B after the collision. [3]

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Sphere B continues to move until it strikes a fixed smooth vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{3}{4}$. When the spheres subsequently collide, A is brought to rest.

(ii) Find the value of e. [7]

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Q17

3 Two uniform small smooth spheres A and B have equal radii and each has mass m. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$. Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is $\frac{1}{3}$.

(i) Show that the speed of B after its collision with the wall is $\frac{5}{18}u$. [4]

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(ii) Find the distance of B from the wall when it collides with A for the second time. [6]

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Q18

3 Three uniform small smooth spheres A, B and C have equal radii and masses m, km and m respectively, where k is a constant. The spheres are moving in the same direction along a straight line on a smooth horizontal surface, with B between A and C. The speeds of A, B and C are $2u$, u and $\frac{4}{3}u$ respectively. The coefficient of restitution between any pair of the spheres is $\frac{1}{2}$. After sphere A has collided with sphere B, sphere B collides with sphere C.

(i) Find an inequality satisfied by k. [5]

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(ii) Given that $k = 2$, show that after B has collided with C there are no further collisions between any of the three spheres. [5]

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Q19

2 A small smooth sphere A of mass m is moving with speed u on a smooth horizontal surface when it collides directly with an identical sphere B which is initially at rest on the surface. The coefficient of restitution between the spheres is e. Sphere B subsequently collides with a fixed vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{1}{2}$. Given that 80% of the initial kinetic energy is lost as a result of the two collisions, find the value of e. [8]

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Q20

5 Three small smooth spheres A, B and C, of masses $5m$, m and km respectively, are at rest on a smooth horizontal surface. The spheres lie in a straight line with B between A and C. Sphere A is projected directly towards B with speed u. The coefficient of restitution between any pair of the spheres A, B and C is $\frac{4}{5}$. Show that the speed of A after the first collision is $\frac{7}{10}u$ and find the speed of B. [3]

Given that the speeds of A and C are equal after the second collision,

(i) show that the value of k is $\frac{20}{7}$, [4]

(ii) find the percentage loss in the kinetic energy of the system as a result of the two collisions. [5]

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Q21

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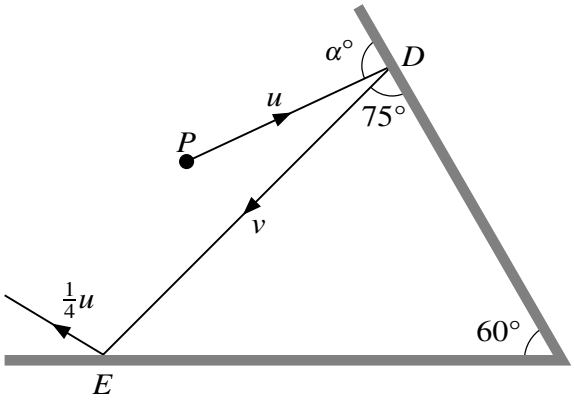

Two smooth vertical walls each with their base on a smooth horizontal surface intersect at an angle of 60°. A small smooth sphere P is moving on the horizontal surface with speed u when it collides with the first vertical wall at the point D. The angle between the direction of motion of P and the wall is $\alpha°$ before the collision and 75° after the collision. The speed of P after this collision is v and the coefficient of restitution between P and the first wall is e. Sphere P then collides with the second vertical wall at the point E. The speed of P after this second collision is $\frac{1}{4}u$ (see diagram). The coefficient of restitution between P and the second wall is $\frac{3}{4}$.

(i) By considering the collision at E, show that $v = \dfrac{\sqrt{2}}{5}u$. [5]

(ii) Find the value of α and the value of e. [5]

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[Turn over

5 Three uniform small smooth spheres A, B and C have equal radii and masses $3m$, $2m$ and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between A and B is e and the coefficient of restitution between B and C is e'. Sphere A is projected directly towards B with speed u. Show that, after the collision between B and C, the speed of C is $\frac{2}{5}u(1+e)(1+e')$ and find the corresponding speed of B. [7]

After this collision between B and C it is found that each of the three spheres has the same momentum. Find the values of e and e'. [5]

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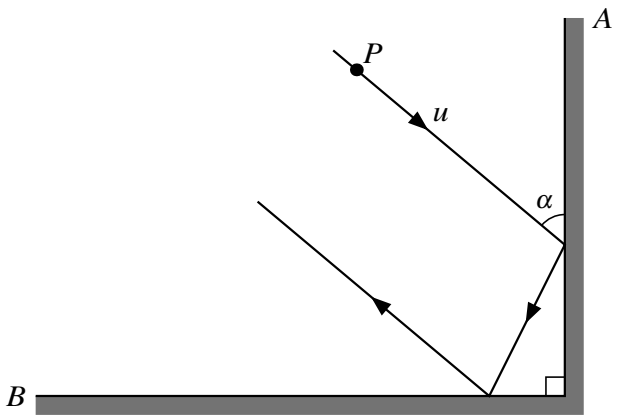
Q23

2



A uniform sphere P of mass m is at rest on a smooth horizontal table. The sphere is projected along the table with speed u and strikes a smooth vertical barrier A at an acute angle α. It then strikes another smooth vertical barrier B which is at right angles to A (see diagram). The coefficient of restitution between P and each of the barriers is e. Show that the final direction of motion of P makes an angle $\frac{1}{2}\pi - \alpha$ with the barrier B and find the total loss in kinetic energy as a result of the two impacts. [7]

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Q24

2 A small uniform sphere A, of mass $2m$, is moving with speed u on a smooth horizontal surface when it collides directly with a small uniform sphere B, of mass m, which is at rest. The spheres have equal radii and the coefficient of restitution between them is e. Find expressions for the speeds of A and B immediately after the collision. [4]

Subsequently B collides with a vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is 0.4. After B has collided with the wall, the speeds of A and B are equal. Find e. [2]

Initially B is at a distance d from the wall. Find the distance of B from the wall when it next collides with A. [4]

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Q25

3 Three small smooth spheres A, B and C have equal radii and have masses m, $9m$ and km respectively. They are at rest on a smooth horizontal table and lie in a straight line with B between A and C. The coefficient of restitution between any pair of the spheres is e. Sphere A is projected directly towards B with speed u. Given that half of the total kinetic energy is lost as result of the collision between A and B, find the value of e. [7]

After B and C collide they move in the same direction and the speed of C is twice the speed of B. Find the value of k. [3]

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Q26

1 Two small smooth spheres A and B have equal radii and have masses m and km respectively. They are moving in a straight line in the same direction on a smooth horizontal table. The speed of A is u and the speed of B is $\frac{2}{3}u$. Sphere A collides directly with sphere B. The coefficient of restitution between the spheres is $\frac{4}{5}$.

(i) Show that the speed of A after the collision is $\dfrac{u(2k+5)}{5(k+1)}$. [5]

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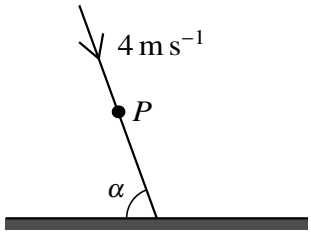
Q27

1 Two smooth spheres A and B, of equal radii and masses $2m$ and m respectively, lie at rest on a smooth horizontal table. The spheres A and B are projected directly towards each other with speeds $4u$ and $3u$ respectively. The coefficient of restitution between the spheres is e. Find the set of values of e for which the direction of motion of A is reversed in the collision. [5]

Q28

2



A small smooth ball P is moving on a smooth horizontal plane with speed $4\,\text{m s}^{-1}$. It strikes a smooth vertical barrier at an angle α (see diagram). The coefficient of restitution between P and the barrier is 0.4. Given that the speed of P is halved as a result of the collision, find the value of α. [5]

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[Turn over

Momentum MS

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Q1

Question	Answer	Marks	Guidance
6(a)	Along line of centres, speeds v_1 and v_2 $mv_1 + mv_2 = mu\cos\alpha - mu\cos\beta$	**M1**	Momentum (condone missing masses).
	$v_2 - v_1 = eu(\cos\beta + \cos\alpha)$	**M1**	Restitution.
	Both correct, masses seen.	**A1**	
	$v_1 = 0$ so A has no speed along line of centres: moves perpendicular to line of centres	**A1**	AG.
		4	
6(b)	$(v_2 = \frac{1}{2}u\cos\alpha = u\cos\beta)$ KE of B after collision is $\frac{1}{2}m\left(v_2^2 + (u\sin\beta)^2\right)$ KE of A after collision = $\frac{1}{2}m(u\sin\alpha)^2$	**M1**	Both components.
	Add both KEs and equate to $\frac{3}{4}mu^2$	**M1**	
	Simplify to equation in $\sin\alpha$	**M1**	
	$\sin\alpha = \frac{1}{\sqrt{2}}$, $\alpha = 45°$	**A1**	
		4	

Q2

Question	Answer	Marks	Guidance
6(a)	Let velocities of A and B along line of centres after collision be v_1 and v_2. $mv_1 + kmv_2 = mu\cos\theta$.	**M1**	Momentum, must include m, allow cos/sin mix.
	$v_2 - v_1 = \frac{1}{3}u\cos\theta$	**M1**	Restitution, consistent signs, correct way up.
	Solve: $v_2 = \frac{4u\cos\theta}{3(1+k)}$	**A1**	AG shown convincingly.
		3	
6(b)	$v_1 = \frac{(3-k)u\cos\theta}{3(1+k)}$	**B1**	Or equivalent, may be unsimplified.
	Use velocity of A with both components.	**B1**	$v_1^2 + (u\sin\theta)^2$ seen.
	$\frac{1}{2}kmv_2^2 + \frac{1}{2}m\left(v_1^2 + (u\sin\theta)^2\right) = \frac{3}{10}\times\frac{1}{2}mu^2$	**M1**	KE after = 30% KE before (all terms present). M0 if incorrect masses.
	Substitute from part **(a)** and for θ.	**M1**	Eliminate trigonometric terms, must be KE equation, in terms of k only.
	$(3-k)^2 + 16k = 2(1+k)^2$, $k^2 - 6k - 7 = 0$	**M1**	Obtain simplified quadratic equation in k.
	$k = 7$	**A1**	
		6	

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Q3

Question	Answer	Marks	Guidance
7(a)	$u\cos\alpha = v\cos\beta$	**M1**	
	$eu\sin\alpha = v\sin\beta$	**M1**	
	Divide: $\tan\beta = e\tan\alpha$	**A1**	AG. Must see divide OE.
		3	
7(b)	$v\sin\beta = w\cos\gamma \quad (= eu\sin\alpha)$	**M1**	
	$ev\cos\beta = w\sin\gamma \quad (= eu\cos\alpha)$	**M1**	
	Divide: $\tan\gamma = 1/\tan\alpha$: $\gamma = 90° - \alpha$	***A**	
	After second rebound, direction of motion is parallel to initial path.	**DB1**	
		4	
7(c)	Final KE = $\frac{1}{2}m\left((eu\sin\alpha)^2 + (eu\cos\alpha)^2\right) \quad \left(= \frac{1}{2}me^2u^2\right)$	**M1**	Energy expression in terms of u.
	So $\frac{1}{2}me^2u^2 = \frac{1}{9}\times\frac{1}{2}mu^2$ giving $e = \frac{1}{3}$	**A1**	
	Part **(a)** gives $\tan(90-\alpha) = e\tan\alpha$	**M1**	
	So $\tan\alpha = \sqrt{3}$	**A1**	
		4	

Q4

Question	Answer	Marks	Guidance
5(a)	Let speeds of A and B along line of centres after collision be v_1 and v_2 $mv_1 + \frac{3}{2}mv_2 = -\frac{3}{2}mu\cos 60° + mu \quad \left(= \frac{u}{4}\right)$	**M1**	Momentum with masses correct.
	$v_2 - v_1 = -\frac{2}{3}(-u\cos 60° - u) \quad (= u)$	**M1**	Restitution, with consistent signs on LHS.
	$\left(v_1 = -\frac{1}{2}u\right) \qquad v_2 = \frac{1}{2}u$	**A1**	
	Perpendicular to line of centres, speed of B is $u\sin 60° = \frac{\sqrt{3}}{2}u$	**B1**	
	Direction of B is now 60° above line of centres.	**M1**	
	Angle of deflection is 60°.	**A1 FT**	FT (120° – *their* direction of B angle)
		6	
5(b)	KE before = $\frac{1}{2}mu^2 + \frac{1}{2}.\frac{3m}{2}u^2 = \frac{5}{4}mu^2$	**B1**	
	KE after = $\frac{1}{2}m\left(\frac{u}{2}\right)^2 + \frac{1}{2}.\frac{3m}{2}\left(\left(\frac{u}{2}\right)^2 + \left(\frac{\sqrt{3}u}{2}\right)^2\right) \quad \left(= \frac{7}{8}mu^2\right)$	**B1 FT**	FT only *their* speeds from **(a)**
	Loss in KE = $\frac{3}{8}mu^2$	**B1**	
		3	

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Q5

Question	Answer	Marks
6(a)	Let components of velocity (parallel to plane and perpendicular) after impact be (x, y)	
	$y = v\cos\alpha = eu\sin\alpha$	**B1**
	$x = v\sin\alpha = u\cos\alpha$	**B1**
	Divide: $\tan\alpha = \frac{1}{e\tan\alpha} : \tan^2\alpha = \frac{1}{e}$.	**B1**
		3
6(b)	$v^2 = \frac{1}{3}u^2$	**B1**
	$\left(\frac{u\cos\alpha}{\sin\alpha}\right)^2 = \frac{1}{3}u^2$	**M1**
	$(\tan\alpha)^2 = 3$	**M1**
	$\alpha = 60°$	**A1**
	$e = \frac{1}{3}$	**A1**
	Alternative method for **6(b)**	
	KE after impact = $\frac{1}{2}m\left(x^2 + y^2\right) = \frac{1}{2}m\left((u\cos\alpha)^2 + e^2(u\sin\alpha)^2\right)$	**M1**
	From **(a)** $\sin\alpha = 1/\sqrt{(1+e)}$ and $\cos\alpha = \sqrt{e}/\sqrt{(1+e)}$	**B1**
	KE = $\frac{1}{2}mu^2\left(\frac{e}{1+e} + \frac{e^2}{1+e}\right) = \frac{1}{2}mu^2 e$	**A1**
	This is equal to $\frac{1}{3}\times\frac{1}{2}mu^2$ so $e = \frac{1}{3}$	**M1**
	$\tan\alpha = \sqrt{3}$, $\alpha = 60°$	**A1**
		5

Q6

Question	Answer	Marks
5(a)	w be speed of A along line of centres after collision	**M1**
	$\leftarrow \quad mw = -mu\cos\alpha + mu\sin\alpha$	
	$w - 0 = e(u\cos\alpha + u\sin\alpha)$	**M1**
	Rearrange: $\sin\alpha(u - eu) = \cos\alpha(u + eu)$	**M1**
	$\tan\alpha = \frac{1+e}{1-e}$. **AG**	**A1**
		4
5(b)	$\tan\alpha = 2 \Rightarrow e = \frac{1}{3}$	**B1**
	$w = \frac{1}{3}u\left(\frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}}\right) = \frac{u}{\sqrt{5}}$	**M1**
	Speed = $\sqrt{w^2 + (u\sin\alpha)^2}$	**M1**
	= $\sqrt{\frac{u^2}{5} + \frac{4u^2}{5}} = u$	**A1**
		4

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Q7

Question	Answer	Marks	Guidance
6(a)	$mu = mw + 2mv$	**B1**	Momentum equation (with m)
	$v - w = eu$	**B1**	Restitution with consistent signs
	$v = \frac{u}{3}(e+1)$ $w = \frac{u}{3}(1-2e)$	**B1**	Both correct.
		3	
6(b)	Perpendicular to plane: $y = ev\sin\theta$ Parallel to plane: $x = v\cos\theta$	**B1**	Both
	Speed of $B = \sqrt{x^2+y^2} = \sqrt{v^2\left(\left(\frac{4}{5}\right)^2 + \left(\frac{2}{3}.\frac{3}{5}\right)^2\right)}$ $\left(= \frac{2}{\sqrt{5}}v\right)$	**M1**	Speed of B
	KE of $B = \frac{1}{2}.2m\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1**	KE of B in terms of u. $\frac{1}{2}$ and $2m$ needed
	KE of $A = \frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2$ So $\frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2 = \frac{5}{32}.\frac{1}{2}.2m\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1 A1**	Relate the two KEs
	$4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	**M1**	Rearrange and simplify to quadratic
	$1+e = \pm 2(1-2e)$ $e = \frac{1}{5}$, 1	**A1**	Both values
		7	

Q8

Question	Answer	Marks	Guidance
2	Speeds v and w after collision $2mv + mw = 2mu\cos\alpha$	**M1**	Momentum equation with m. Correct masses, allow sin instead of cos
	$w - v = eu\cos\alpha$	**M1**	Restitution, with consistent signs
	$v = \frac{1}{3}u\cos\alpha(2-e) = \frac{1}{3}u.\frac{3}{5}\left(2-\frac{1}{3}\right) = \frac{1}{3}u$	**A1**	
	Square of speed of $A = \left(\frac{1}{3}u\right)^2 + (u\sin\alpha)^2$	**M1**	Uses correct speed perpendicular to motion
	$= \left(\frac{1}{3}u\right)^2 + \left(\frac{4}{5}u\right)^2$ Speed $= \frac{13}{15}u$ $(= 0.867u)$	**A1**	
		5	

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Q9

Question	Answer	Marks	Guidance
3(i)	$mv_A + 4mv_B = 4mu - 4mu$ $[v_A + 2v_B = 0]$ (AEF)	M1	Use conservation of momentum for A and B (m may be omitted)
	$v_B - v_A = e(2u + u)$ $[v_B - v_A = 3eu]$	M1	Use Newton's restitution law with consistent LHS signs
	$v_A = -2eu$ *and* $v_B = eu$	A1	Combine to find v_A and v_B (A0 if directions unclear)
		3	
3(ii)	$[4mv_B'] + mv_C = 4mv_B - (4/3)\,mu$ (AEF)	M1	Use conservation of momentum for B & C (m may be omitted)
	$v_C\,[-v_B'] = e(v_B + 4u/3)$	M1	Use Newton's restitution law
	$4v_B - (4/3)\,u = ev_B + 4eu/3,\ 4e - 4/3 = e^2 + 4e/3$	M1	Combine to find quadratic equation for e using $v_B' = 0$
	$3e^2 - 8e + 4 = 0,\quad e = 2/3\quad [v_C = 4u/3]$	A1	Find value of e, (implicitly) rejecting $e = 2$
		4	

Q10

Question	Answer	Marks	Guidance
3(i)	$mv_A + mv_B = 3mu$ $[or\ 3v_A + v_B = 3u]$ (AEF)	M1	Use conservation of momentum for A and B (correct masses)
	$v_B - v_A = eu$	M1	Use Newton's restitution law with consistent LHS signs
	$v_A = \frac{1}{4}(3 - e)\,u$ $[\,v_B = \frac{3}{4}(1 + e)\,u\,]$	A1	Combine to find speed of A
	$w_B + v_C = v_B$ *and* $v_C - w_B = ev_B$ (AEF)	M1	Use conservation of momentum for B and C *and* Newton's restitution law with consistent LHS signs
	$w_B = \frac{1}{2}(1 - e)\,v_B$ *and* $v_C = \frac{1}{2}(1 + e)\,v_B$ $w_B = (3/8)(1 - e^2)\,u$ *and* $v_C = (3/8)(1 + e)^2\,u$ aef	A1 A1	Combine to find w_B *and* v_C in terms of v_B (may be implied) and with v_B replaced (note: $v_C = \frac{3}{8}e^2u + \frac{3}{4}eu + \frac{3}{8}u$)
		6	
3(ii)	$\frac{1}{4}(3 - e) = (3/8)(1 + e)^2$	M1	Find equation in e by equating their v_A and their v_C
	$3e^2 + 8e - 3 = 0 = (3e - 1)(e + 3)$	M1	Simplify and solve resulting quadratic eqn for e,
	$e = 1/3$	A1	(implicitly) rejecting $e = -3$
		3	

Q11

Question	Answer	Marks	Guidance
3(i)	$mv_A + 5mv_B = 5mu$ $[v_A + v_B = u]$ and $v_B - v_A = e\,u$	M1	Use consvn. of momentum for A and B and use Newton's restitution law with consistent LHS signs. AEF
	$v_A = \frac{1}{2}(1 - e)\,u$	A1	Combine to verify speed of A. AG
	$v_B = \frac{1}{2}(1 + e)\,u$	A1	Find speed of B
		3	
3(ii)	$5mv_B' + 3mv_C = 5mv_B$ $[5v_B' + 3v_C = 5v_B]$ $v_C - v_B' = ev_B$	M1	Use consvn. of momentum for B and C and use Newton's restitution law with consistent LHS signs. AEF
	$v_B' = (1/8)(5 - 3e)\,v_B$ $[v_C = (1/8)(5 + 5e)\,v_B\,]$	A1	Combine to find v_B' (v_C not reqd as B, C cannot collide again)
	$\frac{1}{2}(1 - e)\,u \leqslant (1/8)(5 - 3e) \times \frac{1}{2}(1 + e)\,u$	M1	Find condition on e using $v_A \leqslant v_B'$
	$3e^2 - 10e + 3 \leqslant 0$	A1	Simplify to a quadratic inequality
	$\frac{1}{3} \leqslant e$	A1	Solve to give a lower bound on e
	$\frac{1}{3} \leqslant e \leqslant 1$	A1	Non-strict inequality
		6	

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Q12

Question	Answer	Marks	Guidance
3(i)	$mv_A = mu - mku$ (AEF)	**M1**	Use conservation of momentum (m may be omitted)
	$v_A = -e(u + ku)$	**M1**	Use Newton's restitution law
	$-e(1+k) = 1-k$, $e = \dfrac{(k-1)}{(k+1)}$ **AG**	**A1**	Combine to find/verify e
		3	
3(ii)	$KE_I - KE_F = 0{\cdot}6\ KE_I$ *or* $0{\cdot}4\ KE_I = KE_F$	**M1**	Attempt to relate initial and final KEs
	$0{\cdot}4 \times \frac{1}{2}mu^2(1+k^2) = \frac{1}{2}mv_A^2 = \frac{1}{2}mu^2(1-k)^2$	**M1**	Substitute for KEs ($\frac{1}{2}m$ may be omitted)
	$0{\cdot}6k^2 - 2k + 0{\cdot}6 = 0$ *or* $3k^2 - 10k + 3 = 0$ (AEF)	**A1**	Simplify to quadratic eqn in k
	$(3k-1)(k-3) = 0$ so solutions are 3 *and* ⅓	**A1**	Find both solutions of quadratic eqn
	[Reject $k = \frac{1}{3}$] so $k = 3$	**A1**	(Implicitly) reject one solution to find possible value of k since $0 < e < 1$ or $v_A < 0$
	$e = \frac{2}{4} = \frac{1}{2}$ (FT on k)	**A1√**	Find single corresponding value of e
		6	

Q13

Question	Answer	Marks	Guidance
2(i)	$4mv_A + mv_B = 4mu$ (AEF)	**M1**	Use momentum (allow m omitted)
	$v_B - v_A = eu$	**M1**	Use Newton's law (M0 if LHS signs inconsistent)
	$v_A = (1/5)(4-e)\,u$ **AG**	**A1**	Combine to verify/find speeds of A and B after collision
	$v_B = (4/5)(1+e)\,u$	**A1**	
		4	
2(ii)	$v_B' = [-]\,¾\,e\,v_B\ [= -(3/5)\,e\,(1+e)\,u]$	**M1**	Relate vel. v_B' of B after colln. with wall to v_B
	$(1/5)(4-e)\,u = (3/5)\,e\,(1+e)\,u$	**M1**	Equate speeds of A and B (ignore sign of v_B' for both M1s)
	$3e^2 + 4e - 4 = 0$, $e = 2/3$	**A1**	Solve resulting quadratic for e, [implicitly] rejecting root –2
		3	
2(iii)	*EITHER:* $4mw_A + mw_B = 3mv_A\ [= 2mu]$ (AEF) $w_B - w_A = e \times 2v_A = (4/3)v_A$ *or* $(8/9)\,u$ (AEF) $[w_A = ⅓\,v_A = (2/9)\,u]$, $w_B = (5/3)v_A$ *or* $(10/9)\,u$ *OR:* Momentum before colln. is $3mv_A = 2mu > 0$, so after colln. momentum *or* speed of $B > 0$ $w_B > 0$ so B collides again with barrier	**M1 A1**	Use momentum (allow m omitted) with $v_A = -v_B'\ [= ⅔\,u]$ Use Newton's law (M0 if LHS signs inconsistent) Combine to find velocity of B after final collision Allow any similar valid argument Allow $v_A > 0$ by [implicit] inspection
		2	

Q14

Question	Answer	Marks	Guidance
2(i)	$mv_A + 2mv_B = 5mu - 4mu = mu$ (AEF)	**M1**	Use momentum (allow m omitted)
	$v_B - v_A = e(u+2u) = 3eu$	**M1**	Use Newton's law (M0 if LHS signs inconsistent)
	$v_A = (u/7)(1-6e)$	**A1**	Combine to find/verify speeds of A and B after colln.
	$v_B = (u/7)(1+15e)$ AG	**A1**	(ignore signs)
		4	

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2(ii)	$(u/7)(1 - 6e) = -\frac{1}{2}u$, $e = \frac{3}{4}$ *or* 0.75	**M1A1**	Combine to find e from $v_A = -\frac{1}{2}u$ (M0 if dirn. of motion not reversed)
		2	
2(iii)	$KE_A = \frac{1}{2} \times 5m\{u^2 - (\frac{1}{2}u)^2\}$ *and* $[= (15/8)mu^2]$ $KE_B = \frac{1}{2} \times 2m\{(2u)^2 - (7u/4)^2\}$ $[= (15/16)mu^2]$	**M1A1**	Find loss of KE for A and B
	$KE_A / KE_B = (15/8)/(15/16) = 2{:}1$ *or* 2/1 *or* 2	**A1**	Combine to find ratio
		3	

Q15

Question	Answer	Marks	Guidance
2(i)	$mv_A + mv_B = 2mu$ (AEF)	**M1**	Use momentum (allow m omitted)
	$v_B - v_A = \frac{2}{3}u$	**M1**	Use Newton's law (M0 if LHS signs inconsistent)
	$v_A = 4u/9$, $v_B = 10u/9$	**A1, A1**	Combine to find speeds of A and B after collision
		4	
2(ii)	$w_B = [-]\frac{1}{2}v_B$ $[= \pm 5u/9]$	**M1**	Relate speed w_B of B after colln. with wall to v_B (ignore sign)
	EITHER: $(d - x)/v_A = d/v_B + x/w_B$ (AEF)	**M1**	*EITHER:* Equate times in terms of dist. x from wall to 3rd colln.
	$(d - x)/4 = d/10 + x/5$, $x = d/3$	**M1A1**	Substitute for speeds to solve for x
	$t = (d - x)/v_A = (2d/3)/(4u/9) = 3d/2u$	**A1**	and hence find reqd. time t
	OR: $x_A = (d/v_B)v_A = (9d/10u)/(4u/9) = 2d/5$	**(M1**	*OR:* Find dist. x_A moved by A when B reaches wall
	$t = d/v_B + (d - x_A)/(v_A + w_B)$ $= d/(10u/9) + (3d/5)/(4u/9 + 5u/9)$ $= 9d/10u + 3d/5u$	**M1A1**	Find t by adding times to and from wall (or equivalent method)
	$= 3d/2u$	**A1)**	
		5	

Q16

Question	Answer	Marks	Guidance
3(i)	$mv_A + mv_B = 3mu$, $v_B - v_A = eu$ (AEF)	**M1**	Use momentum <u>and</u> Newton's law (**M0** if inconsistent LHS signs; allow $3v_A + v_B = 3u$)
	$v_A = \frac{1}{4}(3 - e)u$, $v_B = \frac{3}{4}(1 + e)u$	**A1, A1**	Combine to find velocities of A and B after colln. (signs must be consistent with chosen direction)
	Total:	**3**	
3(ii)	$v_B' = -\frac{3}{4}v_B$ $[= -(9/16)(1 + e)u]$ (AEF)	**B1**	Relate velocity v_B' of B after colln. with wall to v_B
	$[3mV_A +]\ mV_B = 3mv_A + mv_B'$ $[V_B = 3(9 - 7e)u/16]$	**M1**	Use momentum (allow m omitted and $V_A = 0$)
	$V_B\ [-V_A] = -e(v_B' - v_A)$ $[V_B = e(21 + 5e)u/16]$	**M1**	Use Newton's law
	EITHER: $[4V_A =]\ (3 - e)v_A + (1 + e)v_B' = 0$ $\frac{1}{4}(3 - e)^2 - (9/16)(1 + e)^2 = 0$ (AEF)	**(M1 A1)**	Eliminate V_B with $V_A = 0$ and substitute for v_A and v_B'
	OR: $3(9 - 7e) = e(21 + 5e)$	**(M1 A1)**	
	$5e^2 + 42e - 27 = 0$, $e = 3/5$ *or* 0·6	**M1 A1**	Form and solve quadratic for e, rejecting root –9
	Total:	**7**	

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Q17

Question	Answer	Marks	Guidance
3(i)	$mv_A + mv_B = mu$ (AEF)	*M1	Use conservation of momentum (allow $v_A + v_B = u$)
	$v_B - v_A = \frac{2}{3} u$	*M1	Use Newton's restitution law (consistent LHS signs)
	$v_B = 5u/6$	A1	Combine to find v_B
	$w_B = \frac{1}{3} v_B = 5u/18$ AG	B1	Verify speed w_B of B after collision with wall (ignore sign)
	Total:	4	
3(ii)	$v_A = u / 6$	DA1	Find v_A (dependent on above *M1 *M1)
	EITHER: $(d - x) / v_A = d / v_B + x / w_B$ (AEF)	(M1 A1	*EITHER:* Equate times in terms of reqd. distance x
	$6(d - x) = 1{\cdot}2\, d + 3{\cdot}6\, x$	M1 A1)	Substitute for speeds to formulate an eqn. in x
	OR: $x_A = (d/v_B)\, v_A = (6d/5u)\, u/6 = 0{\cdot}2\, d$	(M1	*OR:* Find dist. x_A moved by A when B reaches wall
	$t_2 = (0{\cdot}8\, d) / (v_A + w_B) = 9d/5u$	M1 A1	Find remaining time t_2
	$y_A = v_A\, t_2 = 0{\cdot}3\, d$ *or* $y_B = w_B\, t_2 = 0{\cdot}5\, d$	A1)	Find remaining distance moved by A or B
	OR2: $x_A = (d/v_B)\, v_A = (6d/5u)\, u/6 = 0{\cdot}2\, d$	(M1	*OR2:* Find dist. x_A moved by A when B reaches wall
	$(0{\cdot}8\, d - x) / v_A = x / w_B$ *or* $0{\cdot}8\, d/(v_A + w_B) = x/w_B$	M1 A1	Equate remaining times to formulate an eqn. in x
	$4{\cdot}8\, d - 6\, x = 3{\cdot}6\, x$ *or* $1{\cdot}8\, d = 3{\cdot}6\, x$	A1)	
	$x = \frac{1}{2} d$	A1	Find x
	Total:	6	

Q18

Question	Answer	Marks	Guidance
3(i)	$mv_A + kmv_B = 2mu + kmu$ (AEF) $[v_A + kv_B = 2u + ku]$	M1	Use conservation of momentum for A & B (allow omission of m in all momentum eqns)
	$v_B - v_A = \frac{1}{2} (2u - u)$ $[= \frac{1}{2} u]$	M1	Use Newton's restitution law with consistent LHS signs
	$v_B = u\,(2k + 5) / 2(k + 1)$ *or* $u\,(k + 5/2) / (k + 1)$ (AEF)	A1	Combine to find v_B
	$[v_A = u\,(k + 4) / 2(k + 1)]$ $v_B > 4u/3$ if $k < 7/2$	M1 A1	Find inequality for k from speeds of B and C after 1st collision
		5	
3(ii)	$kmw_B + mv_C = kmv_B + m(4u/3)$ (AEF) $[2w_B + v_C = 2(3u/2) + 4u/3$ $= 13u/3$ when $k = 2]$	M1	Use conservation of momentum for B & C
	$v_C - w_B = \frac{1}{2} (v_B - 4u/3)$ $[= u/12]$ $(k + 1)\, w_B = (k - \frac{1}{2})\, v_B + 2u$	M1	Use Newton's restitution law with consistent LHS signs Combine to find w_B
	$3\, w_B = (3/2)\, v_B + 2u$ with $v_B = 3u/2$, so $w_B = 17u/12$	*A1	when $k = 2$
	$v_A = u$, $v_A < w_B$	DB1	Verify no further collisions between A and B
	EITHER: $(k + 1)\, v_C = (3k/2)\, v_B + (2 - k)\, (2u/3)$ $3\, v_C = 3\, v_B$ with $v_B = 3u/2$ so $v_C = 3u/2 > w_B$	(DB1)	*EITHER:* Find v_C and verify no further collisions between B and C
	OR: B and C cannot meet again since they move apart after colliding (AEF)	(DB1)	*OR:* State explicitly that no further collisions between B and C
		5	

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Q19

Question	Mark Scheme Details		Part Mark	Total
2	Use momentum and Newton's law, e.g.: (M0 for inconsistent signs; allow $v_A + v_B = u$)	$mv_A + mv_B = mu$, $v_B - v_A = eu$	**M1**	
	Combine to find v_A and v_B (may be implied):	$v_A = \pm(1 - e)\,u/2$, $v_B = \pm(1 + e)\,u/2$	**A1, A1**	
	Relate speed w_B of B after colln. with wall to v_B, e.g.:	$w_B = \pm\frac{1}{2}\,v_B$ [$= \pm(1 + e)\,u/4$]	**B1**	
	Relate KEs before and after (AEF): (Treat error in 1/5 as M1 A0)	$\frac{1}{2}\,mu^2 / 5 = \frac{1}{2}\,mv_A^2 + \frac{1}{2}\,mw_B^2$	**M1 A1**	
	Find e:	$1/5 = (1 - e)^2/4 + (1 + e)^2/16$ $25e^2 - 30e + 9 = (5e - 3)^2 = 0$		
		$e = 3/5$ *or* 0·6	**M1 A1**	[8]

Q20

Question	Mark Scheme Details		Part Mark	Total
5	For A & B use conservation of momentum, e.g.: (m may be omitted here and below)	$5mv_A + mv_B = 5mu$	**M1**	
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = (4/5)\,u$	**M1**	
	Combine to find/verify v_A and find v_B, e.g.:	$6v_A = (5 - 4/5)\,u$, $v_A = 7u/10$ **A.G.** *and* $v_B = 3u/2$	**A1**	[3]
(i)	For B & C use conservation of momentum, e.g.:	$mv_B' + kmv_C = mv_B$	**M1**	
	Use Newton's law of restitution (consistent signs):	$v_C - v_B' = (4/5)\,v_B$	**M1**	
	Combine to find k using $v_C = v_A$, e.g.:	$(k + 1)\,v_C = (1 + 4/5)\,v_B$ $k + 1 = 27/7$, $k = 20/7$ **A.G.**	**M1 A1**	[4]
(ii)	Find v_B' (may be found earlier):	$v_B' = -u/2$	**B1**	
	EITHER: Find final KE, E_{final}, of 3 particles:	$\frac{1}{2}\,5mv_A^2 + \frac{1}{2}\,mv_B'^2 + \frac{1}{2}\,kmv_C^2$ $= \frac{1}{2}\,mu^2\,(49/20 + 1/4 + 7/5)$ $= (41/20)\,mu^2$ *or* 2·05 mu^2	**M1** **A1**	
	OR: Find 1st loss/gain in KE (M1 for either):	$L_1 = \frac{1}{2}\,5mu^2 - \frac{1}{2}\,5mv_A^2 - \frac{1}{2}\,mv_B^2$	**(M1)**	
	Find 2nd loss/gain in KE (A1 for both):	$L_2 = \frac{1}{2}\,mv_B^2 - \frac{1}{2}\,mv_B'^2 - \frac{1}{2}\,kmv_C^2$ [$L_1 = (3/20)\,mu^2$, $L_2 = (3/10)\,mu^2$]	**(A1)**	
	Find loss in KE (ignore sign for M1)	$\frac{1}{2}\,5mu^2 - E_{final}$ *or* $L_1 + L_2$ [$= (9/20)\,mu^2$]	**M1**	
	Hence find percentage loss in KE:	$100\,(9/20)\,mu^2 / (\frac{1}{2}\,5mu^2) = 18\%$	**A1**	[5]

Q21

2 (i)	*EITHER:* Find comps. of speed after colln. at E:	$v \cos 45°$ // to wall *and* $\frac{3}{4}\,v \sin 45°$ ⊥ to wall	**M1 A1**	
	Relate v to u, or v^2 to u^2:	$\sqrt{\{(v/\sqrt{2})^2 + (\frac{3}{4}\,v/\sqrt{2})^2\}} = \frac{1}{4}\,u$ $(5/4\sqrt{2})\,v = \frac{1}{4}\,u$ $v = (\sqrt{2}/5)\,u$ A.G	**M1** **A1** **A1**	
	OR: Relate angle β after colln. to u, v:	$\frac{1}{4}\,u \cos\beta = v \cos 45°$ *and* $\frac{1}{4}\,u \sin\beta = \frac{3}{4}\,v \sin 45°$	**(M1 A1)**	
	Find tan β, or β:	$\tan\beta = \frac{3}{4}$ *or* $\beta = 36·9°$	**(A1)**	
	Eliminate β from either eqn. above, e.g.:	$\frac{1}{4}\,u \times (4/5) = v/\sqrt{2}$ $v = (\sqrt{2}/5)\,u$ A.G.	**(M1)** **(A1)**	[5]
(ii)	Relate comps. of speed // to wall after colln. at D:	$v \cos 75° = u \cos\alpha$	**M1**	
	Find cos α:	$\cos\alpha = (\sqrt{2}/5) \cos 75°$ [= 0·0732]	**A1**	
	Find α:	$\alpha = 85·8°$ *or* 1·50 rads	**A1**	
	Relate comps. of speed ⊥ to wall after colln. at D:	$v \sin 75° = eu \sin\alpha$	**M1**	
	Find e:	$e = (\sqrt{2}/5) \sin 75° / \sin\alpha$ *or* $= \tan 75° / \tan\alpha = 0·274$	**A1**	[5]

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Q22

5	For A & B use conservation of momentum, e.g.: (m may be omitted here and below)	$3mv_A + 2mv_B = 3mu$	M1		
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	M1		
	Combine to find v_B :	$v_B = 3(1 + e)\, u/5$	A1		
	For B & C use conservation of momentum, e.g.:	$2mv_B' + mv_C = 2mv_B$	M1		
	Use Newton's law of restitution (consistent signs):	$v_C - v_B' = e' v_B$	M1		
	Combine to find v_C and v_B':	$v_C = 2(1 + e')\, v_B /3$			
		$= 2(1 + e)\,(1 + e')\, u/5$ A	A1		
		$v_B' = (2 - e')\, v_B /3$			
		$= (1 + e)\,(2 - e')\, u/5$	A1	7	
	Find ratios or values of v_A, v_B', v_C from momentum:	$3v_A = 2v_B' = v_C$ [$= u$]	B1		
	Find e from first collision eqns, e.g.:	$v_A = (3 - 2e)\, u/5 = u/3$			
	(or find e' and then use $3v_A = 2v_B'$)	*or* $v_B = \frac{1}{2}(3u - u)$ *or* $(\frac{1}{3} + e)\, u$			
		$= 3(1 + e)\, u/5$, $e = \frac{2}{3}$	M1 A1		
	Find e' from second collision eqns, e.g.:	$2v_B' = v_C$ so $2(2 - e') = 2(1 + e')$			
		or $v_C = 2(1 + \frac{2}{3})\,(1 + e')\, u/5 = u$			
		or $v_B' = (1 + \frac{2}{3})\,(2 - e')\, u/5 = u/2$			
		$e' = \frac{1}{2}$	M1 A1	5	**12**

Q23

2	*EITHER:* Find/ state comps. of speed after 1st colln.:	$u \cos\alpha$ // to A *and* $eu \sin\alpha \perp$ to A	M1 A1		
	Find/ state comps. of speed after 2nd colln.:	$eu \sin\alpha$ // to B *and* $eu \cos\alpha \perp$ to	A1		
	Find final angle with barrier B: **AG**	$\tan^{-1}(eu \cos\alpha / eu \sin\alpha) = \frac{1}{2}\pi - \alpha$	A1		
	OR: Relate angle β_1 after 1st colln. to α:	$v_1 \sin\beta_1 = eu \sin\alpha$			
		$v_1 \cos\beta_1 = u \cos\alpha$			
		$\tan\beta_1 = e \tan\alpha$	(M1 A1)		
	Relate angle β_2 after 2nd colln. to β_1:	$v_2 \sin\beta_2 = ev_1 \cos\beta_1$			
		$v_2 \cos\beta_2 = v_1 \sin\beta_1$			
		$\tan\beta_2 = e / \tan\beta_1$	(A1)		
	Find final angle β_2 with barrier B:	**AG** $\tan^{-1}(e / e \tan\alpha) = \frac{1}{2}\pi - \alpha$	(A1)	4	
	EITHER: Find total loss or gain in KE, e.g.:	$\frac{1}{2} m\{u^2 - (eu \sin\alpha)^2 - (eu \cos\alpha)^2\}$	M1 A1		
	Find total loss in KE (A0 if wrong sign):	$= \frac{1}{2} m (1 - e^2) u^2$	A1		
	OR: Find or state final speed u_{final} :	$u_{\text{final}} = e u$	(B1)		
	Find total loss in KE (A0 if wrong sign):	$\frac{1}{2} m(u^2 - u_{\text{final}}^2) = \frac{1}{2} m (1 - e^2) u^2$	(M1 A1)	3	**7**

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Q24

2	For A & B use conservation of momentum,	e.g.:$2mv_A + mv_B = 2mu$ (allow $2v_A + v_B = 2u$)	**M1**		
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	**M1**		
	Combine to find v_A and v_B :	$v_A = (2 - e)\, u/3$, $v_B = 2(1 + e)\, u/3$	**A1, A1**	4	
	Find e from $v_A = \lvert v_B' \rvert$ with $v_B' = [-]\, 0{\cdot}4\, v_B$:	$(2 - e) = 0{\cdot}8\,(1 + e)$, $e = 2/3$	**M1 A1**	2	
	EITHER: Equate times in terms of reqd. distance x:	$(d - x)/ v_A = d/v_B + x/v_B'$ (AEF)	**M1 A1**		
	[speeds need not be found:	$v_A = v_B' = 4u/9$, $v_B = 10u/9$]			
	Use $v_A = v_B' = 0{\cdot}4\, v_B$ to solve for x:	$d - x = 0{\cdot}4\, d + x$, $x = 0{\cdot}3\, d$	**M1 A1**		
	OR: Find dist. moved by A when B reaches wall:				
		$d_A = (d/v_B)\, v_A = 0{\cdot}4\, d$	**(M1 A1)**		
	Find reqd. distance x:	$x = \frac{1}{2}(d - d_A) = 0{\cdot}3\, d$	**(M1 A1)**	4	**10**

Q25

3	Use conservation of momentum, e.g.:	$mv_A + 9mv_B = mu$	M1		
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	M1		
	Relate v_A to v_B using K.E. (A.E.F.):	$\frac{1}{2}mv_A^2 + \frac{1}{2}9mv_B^2 = \frac{1}{4}mu^2$	M1		
	Combine two eqns to find v_A and v_B e.g.:	$v_A = (1 - 9e)u/10$, $v_B = (1 + e)u/10$			
		or v_A , $v_B = -u/2$, u/6 [or $7u/10$, $u/30$]	A1		
	Use in 3rd eqn to find e, e.g.:	$(1 - 9e)^2 + 9(1 + e)^2 = 50$			
	(A0 if finally $\pm\frac{2}{3}$)	$90e^2 = 40$, $e = \frac{2}{3}$	M1 A1	7	
	Use Newton's law of restitution with	$v_C = 2v_B'$, e.g.: $v_C - v_B' = ev_B$, $v_B' = \frac{2}{3}v_B$	B1		
		[$v_B = u/6$, $v_B = u/9$, $v_C = 2u/9$]			
	Use conservation of momentum to find k:	$9mv_B' + kmv_C = 9mv_B$			
		$9v_B' + 2kv_B' = 13{\cdot}5v_B'$, $k = 9/4$	M1 A1	3	**10**

Q26

1 (i)	$mv_A + kmv_B = mu + \frac{2}{3}kmu$			
	or $v_A + kv_B = u(1 + \frac{2}{3}k)$ B1			
	$v_A - v_B = -(4/5)(u - \frac{2}{3}u)$			
	or $v_A - v_B = -4u/15$ M1 A1			
	$(1 + k)v_A = u(1 + \frac{2}{3}k - 4k/15)$			
	$v_A = u(2k + 5)/5(k + 1)$ **A.G.** M1 A1			
	[$v_B = u(10k + 19)/15(k + 1)$]	5		

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Q27

1	Use conservation of momentum, e.g.:	$2mv_A + mv_B = 8mu - 3mu$	B1		
	Use restitution (must be consistent with prev. eqn.):	$v_A - v_B = -e\ (4u + 3u)$	B		
	Solve for v_A (or $3v_A$):	$v_A = \frac{1}{3}(5-7e)u$	M1		
		$[v_B = \frac{1}{3}(5+14e)u\ ]$			
	Find lower limit on e for which $v_A < 0$:	$5 - 7e < 0, e > \frac{5}{7}$ *or* 0·714	A1	5	**[5]**

Q28

2	Find speed component along barrier:	$V\cos\beta = 4\cos\alpha$	B1		
	Find speed component normal to barrier:	$V\sin\beta = 0{\cdot}4 \times 4\sin\alpha$	B1		
	Find β by eliminating α with $V = 2$:	$V^2 = 2^2 = 1{\cdot}6^2\sin^2\alpha + 16\cos^2\alpha$	M1		
		$1 - \sin^2\alpha + 0{\cdot}16\sin^2\alpha = 0{\cdot}25$			
		$\sin^2\alpha = \frac{0.75}{0.84} = \frac{25}{28} = 0{\cdot}8929$			
		or $\cos^2\alpha = \frac{3}{28} = 0{\cdot}1071$			
		$\alpha = 1{\cdot}24$ rad *or* 70·9°	M1 A1	5	**[5]**

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